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THE MANCHESTER COLLEGE OF SCIENCE AND TECHNOLOGY

Department of Textile Technology

MECHANICAL BEHAVIOUR OF TWISTED YARNS

2. Geometric Structure and form of yarns

By

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This final technical report has been taken from the full report of the three years' investigation into the subject of the geometric structure and form of yarns, and consequently the numbering of the chapters and pages does not start from unity.

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PART II

GEOMETRIC STRUCTURE AND FORM OF YARNS

Abstract

Yarns may twist as a cylindrical bundle of filaments or as a ribbon of filaments. In some yarns the ribbon form of twisting is more pronounced than the others. In all the earlier theoretical analyses of the geometry and mechanics of twisted yarns, cylindrical structures have been assumed. In our present work, investigations have been made on the structure and form of yarns which do not twist as a cylindrical bundle.

In the previous two annual reports, studies on the untwisting of colour-coated yarns and their cross-sections were presented. Larger model yarn structures were developed to obtain a clear picture of the mechanism of twisting. A theoretical development on the mechanism of twisting based on the ribbon form was made. The agreement between the theoretical and experimental results was found to be very poor and was due to the neglect of various forces acting on the yarn elements. A further development on the theory has been made since and has been included in this report. The improved theory is found to agree very well with the experimental results.

To understand the mechanism of ribbon form of twisting in actual yarns, ribbon of rubber filaments were twisted on model twisters and their cross-sections examined. It is observed that the ideal wrapped ribbon structure as assumed in the theoretical

analysis is not obtained. Instead a collapsed wrapped structure is obtained.

The theory of ribbon twisting has further been applied to determine the path of a filament in the yarn. When a spinning twist is present in the yarn, the filaments do not lie parallel to each other in the ribbon; the presence of a primary migratory cycle in the filament yarns has been explained to be due to the spinning twist. A mathematical relation has been derived for the migration period of a filament assuming that spinning twist is present and the yarn has wrapped ribbon structure.

In the final stages of this work, coloured layers of fibres and filaments have been used to produce yarns on commercial twisting frames. The yarns were then observed under the microscope to show the extent to which wrapped ribbon structure is present in commercial yarns.

CHAPTER 5

GEOMETRY AND MECHANICS OF TWISTING OF RUBBER STRIPS

5.1 Introduction

A theoretical development of the Geometry and Mechanics of Twisting of Rubber Strips is given in this chapter. On account of the complex nature of textile materials, verification of a theory by experiments on actual textile yarns is not easy. The results obtained from the investigation on twisting of rubber strips are compared with corresponding values calculated theoretically, using the measured values of parameters defining the geometry of the system and elastic constants of rubber.

5.2 Geometry of twisted form

The geometrical relations for the twisted form of ribbon can be arrived at by considering it to be of the same type as twisting of a thin metal blade. Figure 5.1 shows the general form of twisted structure. In Figure 5.2(a) it is seen that the lower end of a cylinder whose top end is fixed has been rotated through a certain angle in the direction of the arrow.

Let OO^1 be the axis of the cylinder, and A be any point on the surface of the cylinder at the free end. After twisting, the point A acquires a new position B. Thus the point A has rotated through a certain angle with respect to a similar point A^1

FIG 5.5

FORMS IN WRAPPED STRUCTURE

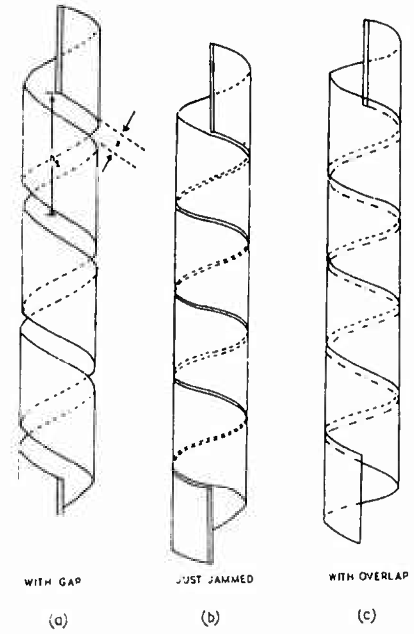


FIG 5.2

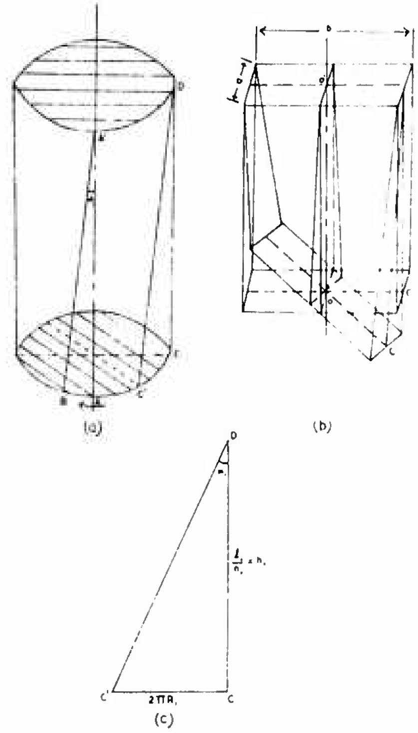
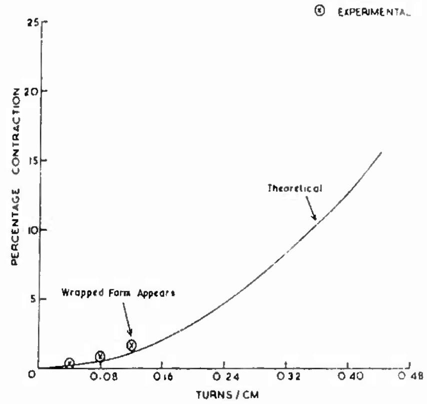
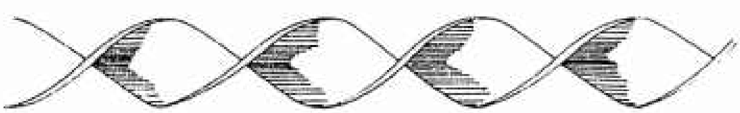


FIG 5.3

CURVES FOR PERCENTAGE CONTRACTION IN TWISTED FORM

(1cm Wide Ribbon)





TWISTED FORM

FIG 5.1

FIG 5.6

GEOMETRY OF WRAPPED STRUCTURE

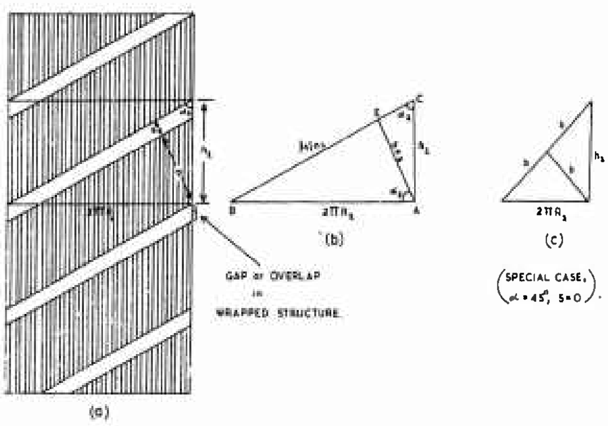
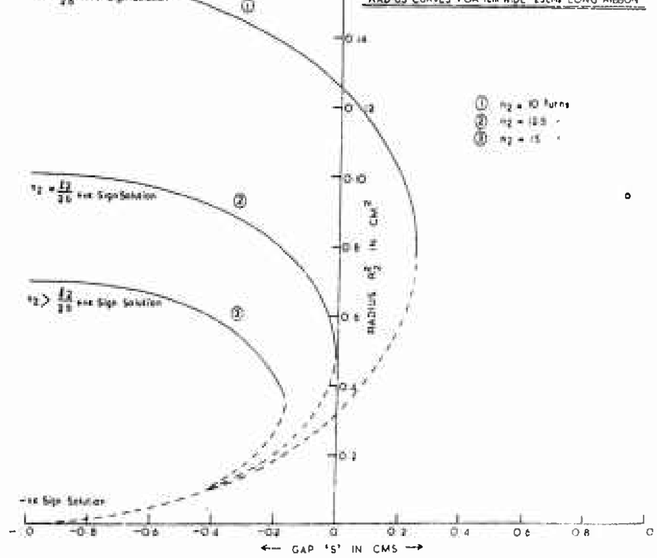


FIG 5.8

RADIUS CURVES FOR 1cm WIDE 25cms LONG RIBBON



on the fixed end of the cylinder. The angle AA^1B is the helix angle. Now if the cylinder is considered to be made up of a number of thin parallel plates then on twisting the cylinder the plates will also be rotated in the similar fashion. If we consider a point C on a plate at the centre of the cylinder then on twisting this point C would have moved to point C^1 and the angle CDC^1 will be equal to the angle AA^1B.

In Figure 5.2(b) twisting of the plate, which was in the centre of the cylinder, has been shown separately. Figure 5.2(c) has been drawn to show the movement of point C to C^1 and D is the point on the fixed end of the plate.

In order to work out the geometry of the twisted part, let

ℓ_1 be the length of strip in twisted part

n_1 be the number of turns in twisted part

R_1 be the radius of the cylinder of which it is a part,

α_1 be the helix angle

The standard expression for the contraction in length of thin rectangular bar during twisting (assuming large deformations in materials like rubber is (24))

$$\epsilon_0 = \frac{\theta^2 b^2}{2\times 12} - \frac{\sigma_0}{Y} \qquad ..... \quad (5)$$

where

ϵ_0 is the percentage contraction in length,

θ is the twist in radians per unit length, b is the width of the rectangular strip,

σ_0 is the longitudinal stress acting on the strip during twisting and Y is the Young's Modulus of the material.

In Fig. 5.3 the calculated curve of percentage contraction in length during twisting of 1 cm. wide ribbon at 5 gms. tension have been plotted. Along with the calculated values of contraction, experimental values have also been plotted. It can be seen that up to 0.12 turns/cm. the agreement between the experimental and theoretical values are very good. The experimental values of contraction beyond 0.12 turns/cm. have not been plotted in Fig. 5.3 because the wrapped part starts forming at that stage and hence the percentage contraction will be much greater. In the equation (5) the width of the ribbon occurs as the squared power, hence the percentage contraction for narrower ribbons will be very small.

Hence, assuming that (a) there is very little deformation in the twisted part, and (b) that the length along strip is equal to the length along the axis, i.e., there is negligible contraction in length (as explained above), the geometrical relations for the twisted part can be arrived at by referring to Fig. 5.2(c).

$$\text{Length in one turn} = CD = \frac{\ell_1}{n_1} = h_1 \text{ (say)}$$

and $$\tan \alpha_1 = \frac{2\pi R_1}{\ell_1/n_1} \qquad \ldots\ldots \quad (5.1)$$

From our assumption (a) :

$$R_1 = b/2$$

$$\therefore \quad \tan \alpha = \frac{\pi b}{\ell_1/n_1} \qquad \ldots\ldots \quad (5.2)$$

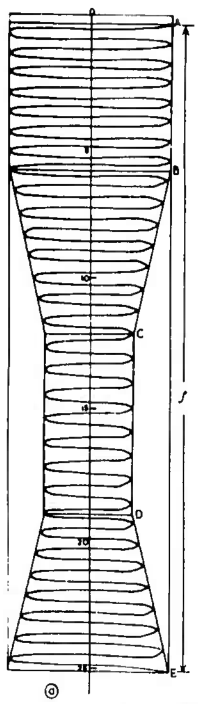
5.3 Geometry of wrapped structure

The form of wrapped structure may be visualised by considering a strip of paper rolled up so that its axis is in one plane as illustrated in Fig. 5.4(a). This is a wrapped structure with a complete overlap of successive turns. If the ends of the paper are now pulled apart, a wrapped structure with only partial overlap is obtained, as in Fig. 5.4(b). Finally, on further extending the chain, a gap between turns will form as in the upper part of Fig. 5.4(c). Thus there are three types of wrapped structure.

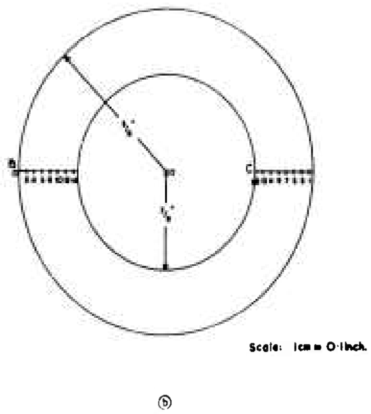
(a) Wrapped structure with overlapping of turns, Fig. 5.5(c). In practice with a strip of rubber of finite thickness the overlap would not be able to take place, and the turns would join together under the condition in which they are trying to overlap.

(b) Wrapped structure with no overlap and no gap, (Fig. 5.5(b)) i.e. a structure which is just jammed.

(c) Wrapped structure with a gap, as shown in Fig. 5.5(a)



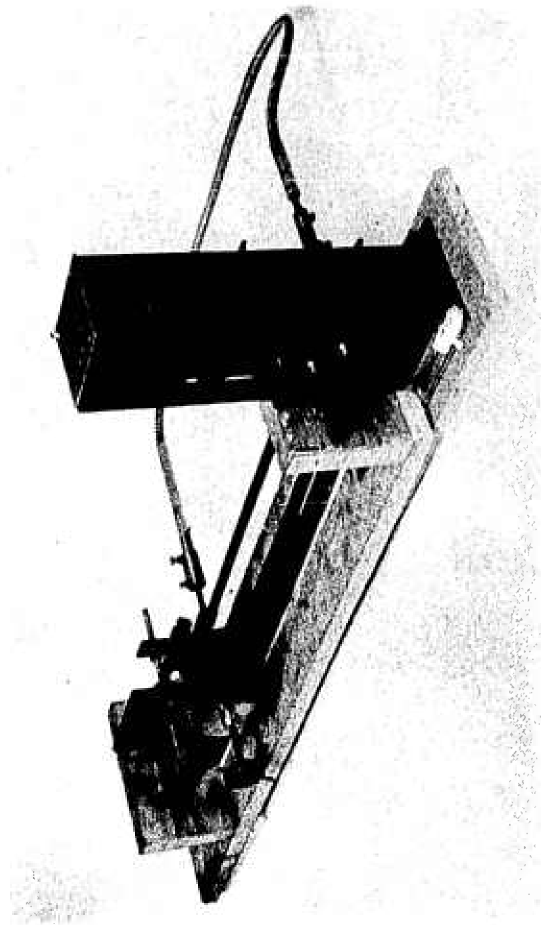
SCALE: Along Length 1" = 0.5cm
Along Radius 0.1" = 0.5cm



Scale: 1cm = 0.1inch.

ⓑ

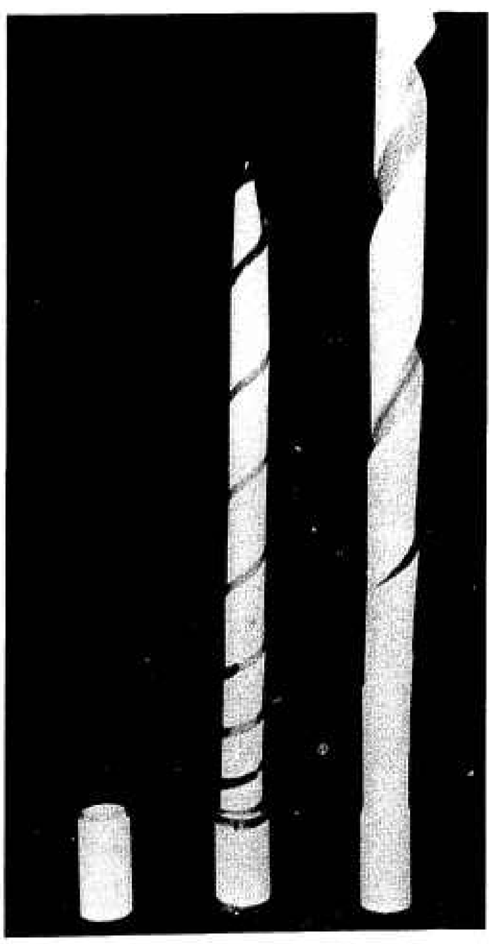
FIG 9.2



MODEL TWISTER.

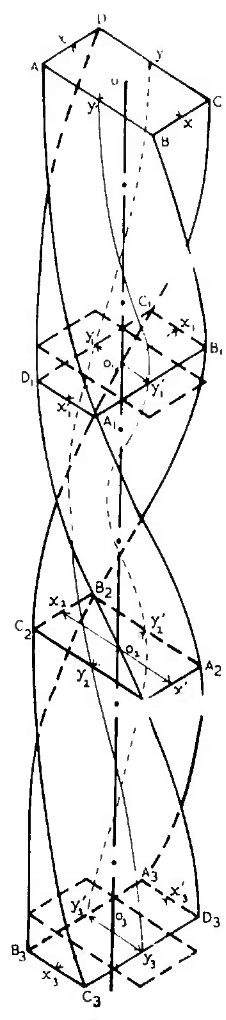
FIG. 6.1

FIG. 5 4



(a) (b) (c)

FIG. 7.4



In order to work out the detailed geometry of the wrapped part, let the length in the wrapped part along the strip $= \ell_2$

the length in the wrapped part along the axis $= L_2$ cms.

the helix angle be $= \alpha_2$

the number of turns in the wrapped part $= n_2$

and the radius of the cylinder $= R_2$

Hence the length in wrapped part along the axis of the structure

$$= \frac{L_2}{n_2} = h_2 .$$

Let S be the separation between the strips perpendicular to the length of the strip as shown in Figure 5.4(a). If there is overlap S will be negative.

Fig. 5.6(a) and (b) has been obtained by cutting the wrapped structure and opening it out into a plane. We see that

$$\tan \alpha_2 = \frac{2\pi R_2}{h_2} \qquad \ldots\ldots \quad (5.3)$$

$$\frac{b + S}{h_2} = \sin \alpha_2 \qquad \ldots\ldots \quad (5.4)$$

$$\frac{h_2}{\ell_2/n_2} = \cos \alpha_2 \qquad \ldots\ldots \quad (5.5)$$

$$\frac{2\pi R_2}{\ell_2/n_2} = \sin \alpha_2 \qquad \ldots\ldots \quad (5.6)$$

$$\frac{b + S}{2\pi R_2} = \cos \alpha_2 \qquad \ldots\ldots \quad (5.7)$$

from (5.6) and (5.7) we get :

$$\frac{(b + S)^2}{4\pi^2 R_2^2} = \cos^2 \alpha_2$$

$$\frac{4\pi^2 R_2^2}{(\ell_2/n_2)^2} = \sin^2 \alpha_2$$

$$\therefore \quad \frac{4\pi^2 R_2^2}{(\ell_2/n_2)^2} = 1 - \frac{(b + S)^2}{4\pi^2 R_2^2} \qquad \ldots\ldots \quad (5.8)$$

or $$16\pi^4 R_2^4 = \left(\frac{\ell_2}{n_2}\right)^2 \left\{4\pi^2 R_2^2 - (b + S)^2\right\}$$

or $$16\pi^4 R_2^4 - 4\pi^2 \frac{\ell_2^2}{n_2^2} R_2^2 + (b + S)^2 \frac{\ell_2^2}{n_2^2} = 0.$$

$$\therefore \quad R_2^2 = \frac{4\pi^2 \frac{\ell_2^2}{n_2^2} \pm \sqrt{16\pi^4 \left(\frac{\ell_2^2}{n_2^2}\right)^2 - 64\pi^4 (b + S)^2 \cdot \frac{\ell_2^2}{n_2^2}}}{2 \cdot 16\pi^4}$$

$$= \frac{4\pi^2 \left[\frac{\ell_2^2}{n_2^2} \pm \frac{\ell_2}{n_2} \sqrt{\frac{\ell_2^2}{n_2^2} - 4(b + S)^2}\right]}{32\pi^4}$$

or $$R_2^2 = \frac{\frac{\ell_2^2}{n_2^2} \pm \frac{\ell_2}{n_2} \sqrt{\frac{\ell_2^2}{n_2^2} - 4(b + S)^2}}{8\pi^2}$$

or $$R_2^2 = \frac{\ell_2^2 \pm \ell_2 \sqrt{\ell_2^2 - 4(b + S)^2 n_2^2}}{8\pi^2 n_2^2} \qquad \ldots\ldots \quad (5.9)$$

Also from Fig. 5.6(b) we find that

$$h_2 = (b + S) \operatorname{cosec} \alpha_2 \qquad \ldots\ldots \quad (5.10)$$

Equation (5.10) gives the general formula for the length of one turn of the structure along the axis and equation (5.9) is the general equation for the radius of the wrapped structure. S can assume positive or negative values or it can be zero. When S has positive values we have wrapped structure with gap, but when S is negative we obtain the wrapped structure with overlapping. The zero value of S corresponds to the jammed structure, but the general form of the equation (5.9) and (5.10) will be used initially in dealing with the mechanics of the wrapped structure.

Equation (5.9) has two solutions because it is in general possible, as illustrated in Fig. 5.7, to have two structures with the same length of strip, the same number of turns, and the same gap (or overlap) between the turns of the strip.

It is interesting to consider some special cases:

(i) Considering the positive sign only in equation (5.9)

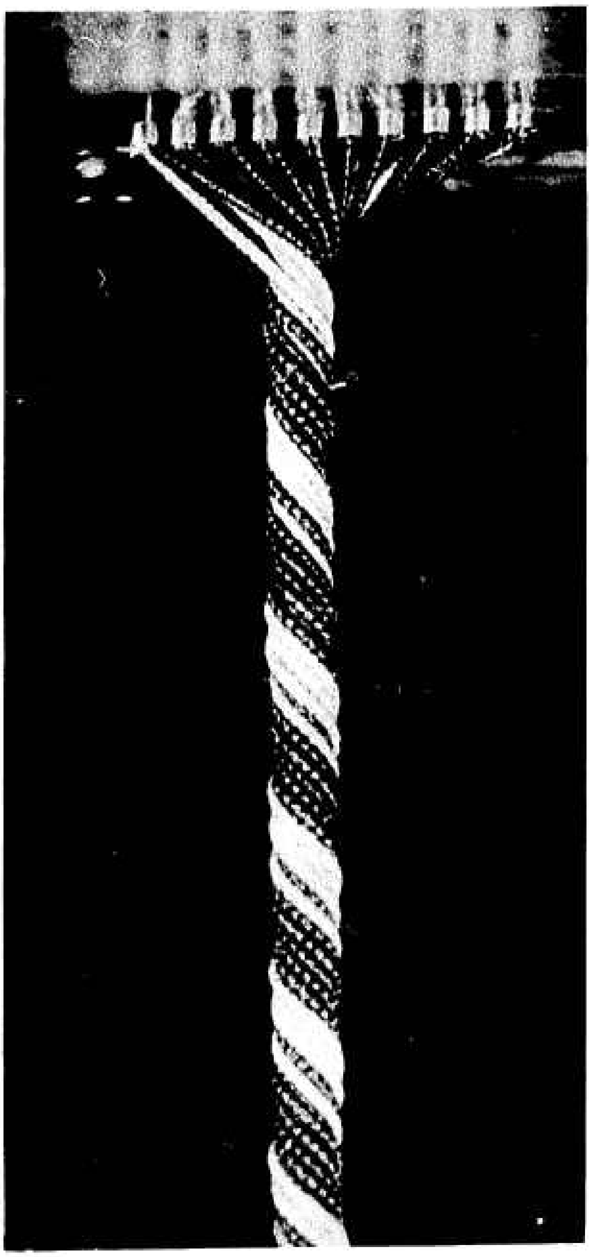
$$R_2^{\,2} = \frac{\ell_2^{\,2} + \ell_2 \sqrt{\ell_2^{\,2} - 4(b+S)^2 n_2^{\,2}}}{8\pi^2 n_2^{\,2}} \qquad \ldots\ldots \quad (5.9a)$$

This indicates a reduction in diameter with the increase in twist.

(ii) When $S = -b$, i.e. there is complete overlap

FIG. 6.11

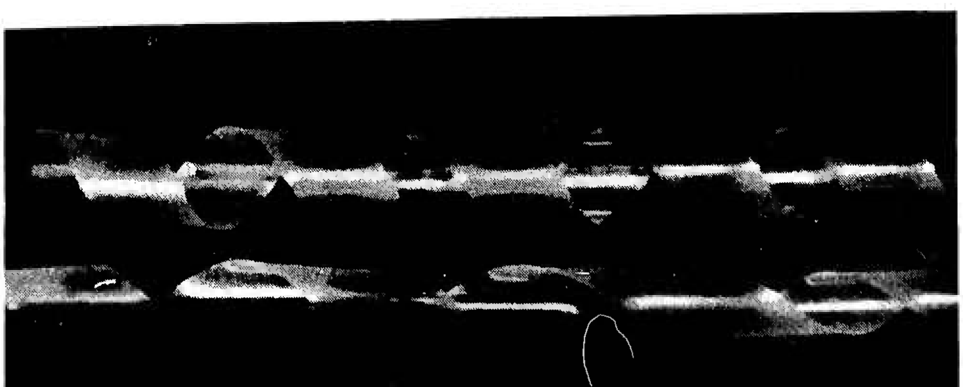
VIEW FROM TOP.



VIEW FROM BOTTOM.

FIG. 5.7

TWO SOLUTIONS OF WRAPPED STRUCTURE.



$$R_2^2 = \frac{\ell_2^2}{4\pi^2 n_2^2} \qquad \ldots\ldots \quad (5.9b)$$

This is correct since the strip is wrapped in one plane Fig. 5.4(a) and its circumference must equal ℓ_2/n_2.

(iii) When $S = 0$, i.e., the structure is just jammed,

$$R_2^2 = \frac{\ell_2^2 \pm \ell_2 \sqrt{\ell_2^2 - 4b^2 n_2^2}}{8\pi^2 n_2^2} \qquad \ldots\ldots \quad (5.9c)$$

Again two solutions are in general possible, corresponding to jamming in tension or compression. In Fig. 5.8 graphs have been plotted at various twists for the radius of the wrapped structure. It will be seen that the two solutions as mentioned above give rise to two different values of radius for the same twist and the same gap 'S' between their edges. The two solutions merge into one at one particular gap for each value of twist.

(iv) General Condition for the two solutions to merge.

From the study of the curves in Fig. 5.8 it will be seen that in case (a) when the number of turns introduced $n_2 < \frac{\ell_2}{2b}$ the two solutions merge into one when the gap S is 0.25 for 1 cm. wide ribbon of length 25 cm. at 10 turns.

(b) when $n_2 = \frac{\ell_2}{2b}$ the two solutions merge together at zero gap and the value of radius is given by

$$R_2^{\,2} = \frac{\ell_2^{\,2}}{8\pi^2 n_2^{\,2}} \qquad \ldots\ldots \quad (5.9d)$$

In this case the geometry of the structure will be as shown in Fig. 5.6c with $\alpha = 45^0$. No structure with gap is possible when $n_2 = \frac{\ell_2}{2b}$.

(c) When $n_2 > \frac{\ell_2}{2b}$ no wrapped structure with gap is possible. The two solutions merge into one for a definite value of overlap depending on the value of n_2.

From the Fig. 5.8 it can be further seen that the -ve sign solution will give an increase in radius of the wrapped structure with twist and also an increase in radius with the gap. This is quite contrary to the observations when a ribbon is twisted under tension. As a matter of fact the above situation can only arise when a strip is twisted under compression. The observed effect of reduction in diameter with increase in twist and increase in gap is explained by the +ve sign solution graphs in Fig. 5.8.

5.4 Mechanics of the twisted part

Let the width of the ribbon be b cms.

thickness of the ribbon be d cms.

and length of the ribbon be ℓ cms.

If the thickness 'd' of the ribbon be very small compared to the width b, then the standard expression for couple (23) required to twist a thin blade, one end of which is fixed, can be used for our purpose

$\therefore$ The couple required to twist a rectangular strip of width b and thickness d will be given by

$$G = C\eta\tau bd^3 \text{ dynes cm.}$$

where η is the shear modulus

τ is the twist in radians per cm.

and C is a constant and is determined by b to d ratio.

or $$G = C\eta\frac{\theta}{\ell}bd^3$$

where θ is the rotation of the free end in radians.

Applying this equation to the twisted part of the structure

we know that $\theta = 2\pi n$,

and $\ell = \ell_1$

$$\therefore \text{ Couple G becomes } = C\eta \cdot \frac{2\pi n_1}{\ell_1} \cdot bd^3$$

$$= 2C\pi\eta bd^3 \cdot \frac{n_1}{\ell_1} \qquad \ldots\ldots (5.11)$$

When b to d ratio is 10 or more $C = \frac{1}{3}$ and hence

$$G = \frac{2}{3}\pi\eta bd^3\frac{n_1}{\ell_1} \qquad \ldots\ldots (5.11a)$$

(will be applicable to the 1 cm. wide ribbon where $\frac{b}{d} = \frac{1}{0.09} \simeq 10$)

Again when b to d ratio is approximately equal to 5 the value of $C = 0.29$

$$\therefore \quad G = 0.58\pi\eta bd^3 \cdot \frac{n_1}{\ell_1} \qquad \ldots\ldots (5.11b)$$

(will be applicable to the 0.5 cm. wide ribbon where
$\frac{b}{d} = \frac{0.5}{0.09} \underline{\Omega} 5$).

Strain Energy

When a length of a bar is twisted through certain angle then work is done on it and this work done is stored in the body as energy. If the body is a perfectly elastic one, then this energy is released on removal of the forces causing this strain. Thus, if G be the couple applied to twist a body through an angle θ, then the work done

$$= \text{Strain Energy} = \frac{1}{2} G\theta$$

In our case of twisting a rectangular strip

$$\text{Strain Energy} = E_T = \frac{1}{2} G\theta$$

$$= \frac{1}{2} \cdot 2C\pi \eta bd^3 \cdot \frac{n_1}{\ell_1} \cdot 2\pi n_1$$

$$= 2C\pi^2 \eta bd^3 \cdot \frac{n_1^2}{\ell_1} \qquad \ldots\ldots \quad (5.12)$$

For 1 cm. wide strip the strain energy

$$E_T = 2 \cdot \frac{1}{3} \pi^2 \eta bd^3 \cdot \frac{n_1^2}{\ell_1} \qquad \ldots\ldots \quad (5.12a)$$

and For 0.5 cm. wide strip the strain energy

$$E_T = 0.58 \pi^2 \eta bd^3 \cdot \frac{n_1^2}{\ell_1} \qquad \ldots\ldots \quad (5.12b)$$

Thus in general

$$E_T = \pi n_1 G \qquad \ldots\ldots \quad (5.13)$$

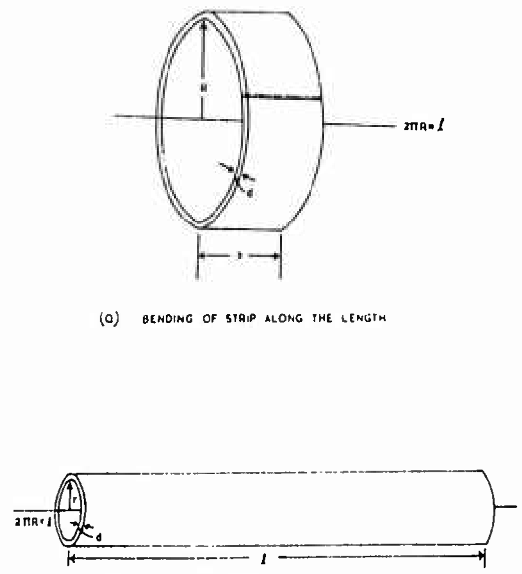
5.5 Mechanics of Wrapped Part

If a long thin strip of rubber or a strip of paper is bent along its length so that two ends of the strip are made to touch each other, then we get a cylindrical shell structure whose depth is equal to the width of the strip and the circumference equal to the length of the strip as shown in Fig. 5.9(a). Again if the same strip of rubber or paper is bent along its width so that the two edges of the strip are made to touch each other then we get another cylindrical shell structure whose depth will be equal to the length of the strip and the circumference equal to the width of the strip, as shown in Fig. 5.9(b). But if we bend the strip in skew by holding two ends of the strip, such that the axis of bending is inclined at a certain angle to the length of the strip, then the type of structure obtained will not be the same as the other two types mentioned above but it will be as shown in Fig. 5.10(a) If bending in the rest of the length of the strip is continued, keeping the axis of bending the same and also the radius of curvature of bending as constant, then we get a structure as shown in Fig. 5.10(b) and (c). This structure resembles the structure of the wrapped part Fig. 5.5(a).

A short section of the wrapped structure is shown in Fig. 5.11. In order to maintain this configuration, torque must be applied along AB and CD to balance the bending moment.

The bending moment M for a rectangular bar when bent into

FIG 5.9



(a) BENDING OF STRIP ALONG THE LENGTH

(b) BENDING OF STRIP ALONG THE WIDTH

FIG 5.10

BENDING OF STRIP AT SKEW

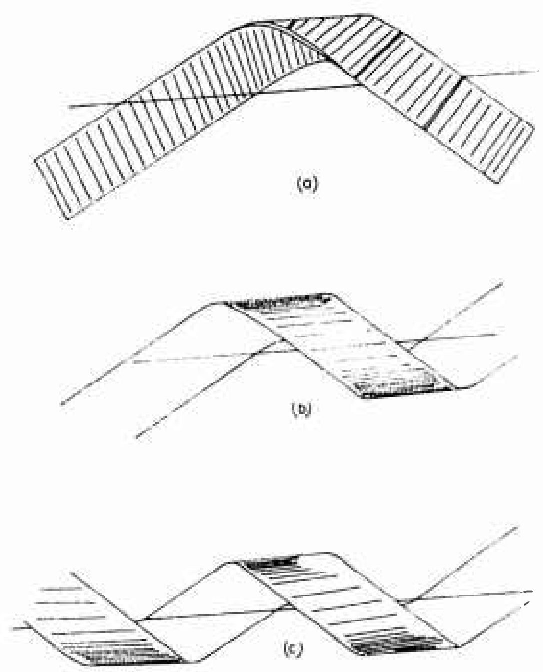


FIG 5.11

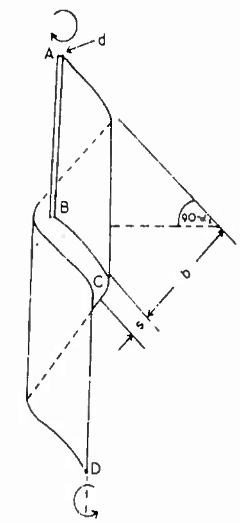


FIG 5.12

BENDING OF A BLADE IN SKEW

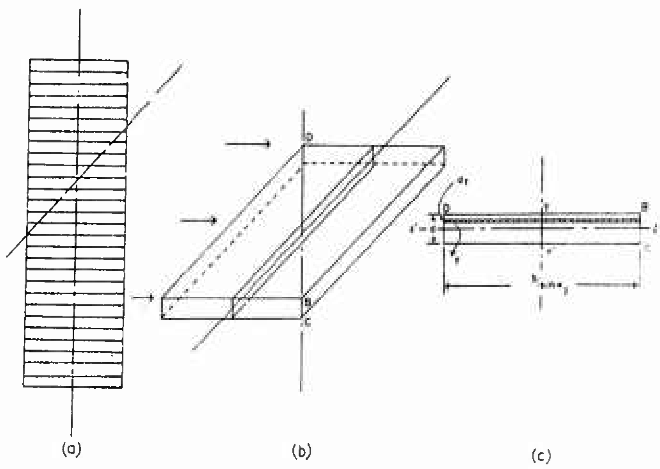


FIG 5.13

ENERGY DUE TO BENDING AT VARIOUS GAPS.

(1cm Wide & 25cms Long Ribbon.)

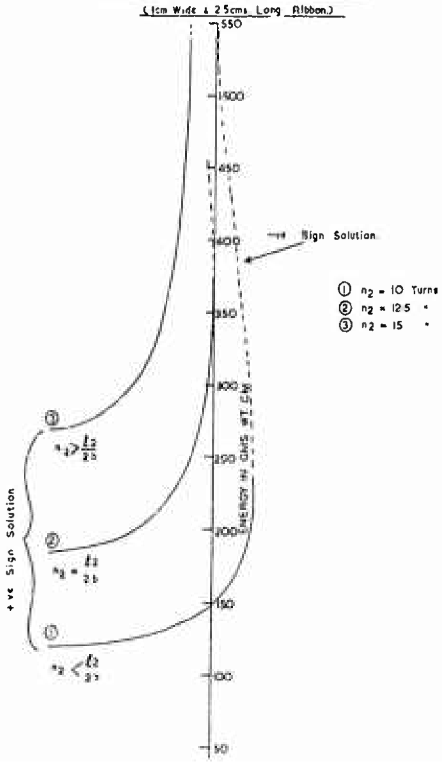
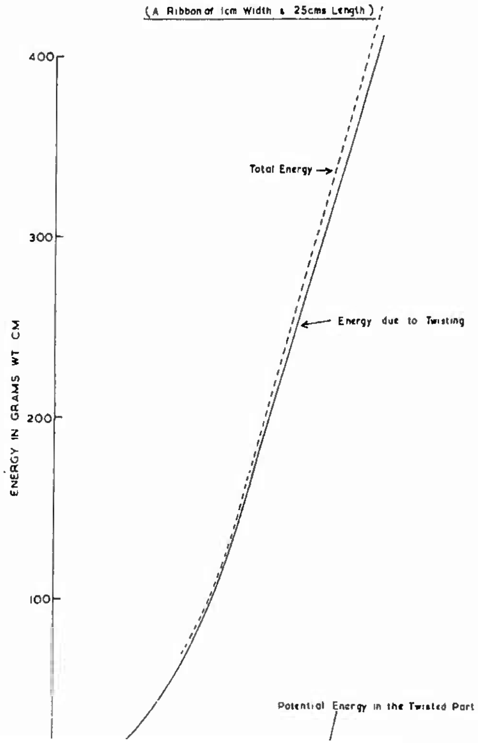


FIG 5.14

ENERGY IN THE TWISTED PART WHEN CONTRACTION IN THE TWISTED PART IS CONSIDERED

(A Ribbon of 1cm Width & 25cms Length)



a cylinder (i.e. considering large deformation) is given by (26) -

$$M = \frac{Y \cdot I}{(1 - \mu^2)\rho}$$

where Y is the Young's modulus of the material

I is the moment of inertia of the bar about the neutral axis, ρ is the radius of the cylinder into which the bar has been bent and μ is the Poisson's ratio for the material.

Applying the same formula to the wrapped structure, we have

$$\rho = R_2 \quad \text{(radius of the wrapped structure)}$$

If ZZ^1 be the netural axis, then from Figure 5.12(c) moment of inertia of the rectangular strip which is the thickness of the rubber along AB in Figure 5.11 is

$$I_{ZZ^1} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \frac{b}{\sin \alpha_2} \cdot dy$$

$$= \frac{1}{12} \cdot \frac{bd^3}{\sin \alpha_2} \qquad \ldots\ldots \quad (5.14)$$

Hence

$$\text{Bending Moment} = M = \frac{1}{12} \cdot \frac{1}{(1 - \mu^2)} \cdot \frac{bd^3}{\sin \alpha_2} \cdot \frac{1}{R_2} \cdot Y \qquad \ldots\ldots \quad (5.15)$$

and

$$\text{Torque} = \Upsilon = \text{Bending Moment } M = \frac{Y \cdot bd^3}{12(1 - \mu^2)} \cdot \frac{1}{\sin \alpha_2} \cdot \frac{1}{R_2} \qquad \ldots\ldots \quad (5.16)$$

Substituting the value of sin α_2 from equation (5.6) in equation (5.16) we get:

$$\Upsilon = \frac{Y \cdot bd^3}{12(1-\mu^2)} \cdot \frac{\ell_2}{2\pi R_2 n_2} \cdot \frac{1}{R_2}$$

$$= \frac{Y \cdot bd^3}{24(1-\mu^2)\pi} \cdot \frac{\ell_2}{n_2} \cdot \frac{1}{R_2^2} \quad \ldots\ldots \quad (5.17)$$

Substituting the value of R_2^2 from equation (5.9) in equation (5.17) we get:

$$\Upsilon = \frac{Y \cdot bd^3}{24(1-\mu^2)\pi} \cdot \frac{\ell_2}{n_2} \cdot \frac{8\pi^2 n_2^2}{\ell_2^2 \pm \ell_2\sqrt{\ell_2^2 - 4(b+S)^2 n_2^2}} \quad \ldots\ldots \quad (5.18)$$

$$\text{or} \quad \Upsilon = \frac{Y \cdot bd^3}{3(1-\mu^2)} \cdot \frac{\pi\, n_2}{\ell_2 \pm \sqrt{\ell_2^2 - 4(b+S)^2 n_2^2}} \quad \ldots\ldots \quad (5.19)$$

The two solutions correspond to the two forms having same values of ℓ_2, n_2, b and S but different values of α_2 and R_2.

Strain Energy

When a strip is bent by an externally applied couple work is done on it. This work is stored in the body in the form of energy, and is known as the strain energy due to bending. Strain energy due to bending is given by half the product of the bending moment and the angle which the ends of the bent strip submit at the centre of the circle of which it is a part. In our case as the bending moment is equal to the torque Υ, the strain energy stored in the wrapped part will be given by

$$E_{wr} = \frac{1}{2} \text{ torque x total twist}$$

$$= \frac{1}{2} \times \Upsilon \times 2\pi n_2$$

$$= \pi \cdot n_2 \cdot \Upsilon \qquad \ldots\ldots \quad (5.20)$$

Special cases

(i) Considering the positive sign in equation (5.19)

$$E_{wr} = \frac{Ybd^3}{3(1-\mu^2)} \cdot \frac{\pi^2 n_2^2}{\ell_2 + \sqrt{\ell_2^2 - 4(b+S)^2 n_2^2}} \qquad \ldots\ldots \quad (5.21a)$$

(ii) now if S = -b

$$E_{wr} = \frac{Ybd^3}{3(1-\mu^2)} \cdot \frac{\pi^2 n_2^2}{2\ell_2} \qquad \ldots\ldots \quad (5.21b)$$

agreeing with the simple equation of planer wrapping

(iii) When $S = 0$

$$E_{wr} = \frac{Ybd^3}{3(1-\mu^2)} \cdot \frac{\pi^2 n_2^2}{\ell_2 \pm \sqrt{\ell_2^2 - 4b^2 n_2^2}} \qquad \ldots\ldots \quad (5.21c)$$

(iv) When $S = 0$ and $n_2 = \frac{\ell_2}{2b}$

$$E_{wr} = \pi^2 \cdot \frac{Ybd^3}{3(1-\mu^2)} \cdot \frac{\ell_2}{4b^2} \qquad \ldots\ldots \quad (5.21d)$$

In Figs. 5.13 curves for the energy due to bending has been plotted against the gap 'S' for various values of twist. If the curves plotted with the +ve solution are observed, it will be seen that when n_2 is less than $\ell_2/2b$, the energy increases as the gap increases. If the ribbon was free to choose a structure due to bending then it would choose a form which will have minimum energy stored in it. When the ribbon has got a finite thickness, it is obvious that it will not be able to overlap and so will not achieve the form with minimum energy occurring with overlap of $S = -b$. If no external tension was applied during wrapping, the ribbon would have taken the form with zero gap which has got less bending energy than the form with a gap. When n_2 is equal to $\ell_2/2b$ there will be only one form possible without overlapping and the energy level position is shown in Fig. 5.13. When n_2 is greater than $\ell_2/2b$ a wrapped structure without overlap is not possible (as shown in Fig. 5.13), and if the ribbon is of finite thickness, distortion of structure will take place at this twist for the tendency of the edges will be to overlap which will be obstructed due to its thickness.

5.6 Potential energy of the system

Energy is defined as the capacity for doing work, and when a stone is at a height of h cm. above the ground its weight has the capacity of doing mgh ergs of work in virtue of its position. It is said to possess energy of position or potential energy.

When a ribbon is twisted the length along the axis of the structure is smaller than the length along the axis of the strip, i.e. twisting is followed with contraction in length. If a tension had been acting on the ribbon when it was being twisted, or if we imagine that a load was hung from one end of the ribbon and twist was imparted from the other end, the load will be lifted up due to the contraction in length. Thus work is being done by the load or the tension during the process of twisting, which is stored as potential energy of the system. If the twist is taken out or the twisting process is reversed, the potential energy is liberated.

In Fig. 5.14 the Potential Energy curve for a 1 cm. wide and 25 cm. long ribbon has been plotted along with the energy curve due to twisting. The total energy curve which is the sum of two energies shows a very small difference from the energy curve due to twisting only and hence for all our future estimation potential energy due to twisting will be neglected.

Potential energy due to the wrapped part

If the length along the axis of the wrapped structure be h_2 cms. for one turn of twist and the length along the ribbon be ℓ_2 cms., then the total contraction in length when n_2 number of turns have been introduced will be:

$$\text{Contraction} = \ell_2 - n_2 h_2 \qquad \ldots\ldots \quad (5.22)$$

Now the length of 1 turn along the axis $= h_2$

$$= \left[\frac{\ell_2^{\,2}}{n_2^{\,2}} - 4\pi^2 R_2^{\,2}\right]^{1/2}$$

$$= \left[\frac{\ell_2^{\,2}}{n_2^{\,2}} - 4\pi^2 \cdot \frac{\ell_2^{\,2} \pm \ell_2\sqrt{\ell_2^{\,2} - 4(b+S)^2 \cdot n_2^{\,2}}}{8\pi^2 n_2^{\,2}}\right]^{1/2}$$

$$= \left\{\frac{\ell_2^{\,2}}{n_2^{\,2}} - \frac{1}{2}\,\frac{\ell_2^{\,2}}{n_2^{\,2}} \cdot \left[1 \pm \sqrt{1 - \frac{4(b+S)^2 n_2^{\,2}}{\ell_2^{\,2}}}\right]\right\}^{1/2}$$

therefore

$$h_2 = \frac{\ell_2}{n_2}\left[1 - \frac{1}{2}\left\{1 \pm \left(1 - 4(b+S)^2\,\frac{n_2^{\,2}}{\ell_2^{\,2}}\right)^{1/2}\right\}\right]^{1/2}$$

$$= \frac{\ell_2}{n_2}\left[\frac{1}{2} \mp \frac{1}{2}\left(1 - 4(b+S)^2\,\frac{n_2^{\,2}}{\ell_2^{\,2}}\right)^{1/2}\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\ell_2}{n_2}\left[1 \mp \left(1 - 4(b+S)^2\,\frac{n_2^{\,2}}{\ell_2^{\,2}}\right)^{1/2}\right]^{1/2} \qquad \ldots\ldots \quad (5.23)$$

therefore contraction in length $= \ell_2 - n_2 h_2$

$$= \ell_2 - \frac{1}{\sqrt{2}}\,\ell_2\left[1 \mp \left\{1 - 4(b+S)^2 \cdot \frac{n_2^{\,2}}{\ell_2^{\,2}}\right\}^{1/2}\right]^{1/2}$$

$$= \ell_2\left[1 - \frac{1}{\sqrt{2}}\left\{1 \mp \left(1 - 4(b+S)^2 \cdot \frac{n_2^{\,2}}{\ell_2^{\,2}}\right)^{1/2}\right\}^{1/2}\right] \text{ cms.} \qquad \ldots\ldots \quad (5.24)$$

Therefore if W be the tension during the twisting process or if W be the weight in gms. hung from the end of the ribbon during twisting:

$$\text{Potential Energy} = W.\ \ell_2 \left[1 - \frac{1}{\sqrt{2}} \left\{ 1 \mp \left(1 - 4(b+S)^2 .\ \frac{n_2^{\,2}}{\ell_2^{\,2}}\right)^{1/2} \right\}^{1/2} \right] \text{ gms. cm.} \qquad \ldots\ldots \quad (5.25)$$

Special cases

(i) Considering the negative sign in equation (5.25) corresponding to the positive sign in equations (5.9) and (5.19) Potential Energy

$$= W\,\ell_2 \left[1 - \frac{1}{\sqrt{2}} \left\{ 1 - \left(1 - 4(b + S)^2 \frac{n_2^{\,2}}{\ell_2^{\,2}}\right)^{1/2} \right\}^{1/2} \right] \text{ gms. cm.} \qquad \ldots\ldots \quad (5.26a)$$

(ii) now if $S = -b$

$$\text{Potential energy} = W\,\ell_2 \qquad \ldots\ldots \quad (5.26b)$$

In the planar wrapped structure, the whole length of the strip is taken up.

(iii) when $S = 0$

$$\text{Potential energy} = W\,\ell_2 \left[1 - \frac{1}{\sqrt{2}} \left\{ 1 - \left(1 - 4\,\frac{b^2 n_2^{\,2}}{\ell_2^{\,2}}\right)^{1/2} \right\}^{1/2} \right] \text{ gms. cm.} \qquad \ldots\ldots \quad (5.26c)$$

(iv) when $S = 0$ and $n_2 = \dfrac{\ell_2}{2b}$

$$\text{Potential energy} = w\,\ell_2 \left(1 - \frac{1}{\sqrt{2}}\right) \qquad \ldots\ldots \quad (5.26d)$$

In Fig. 5.15 the Potential Energy curves for both the solutions have been plotted against gap S for three values of

FIG 5.16

VARIATION OF TOTAL ENERGY IN WRAPPED FORM WITH GAP

(1cm Wide x 25cm Long Ribbon)

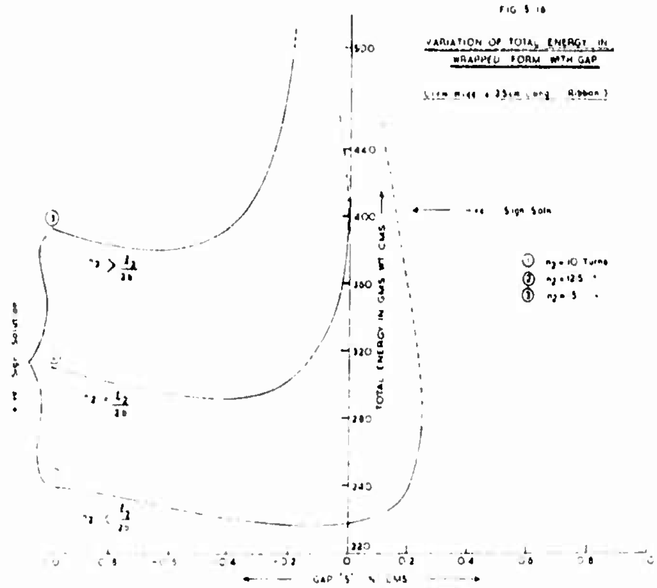


FIG 5.15

POTENTIAL ENERGY CURVES

(1cm Wide x 25cm Long Ribbon)

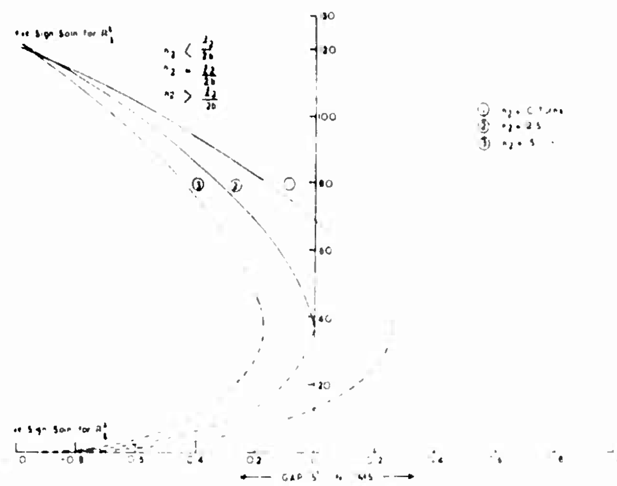


FIG 5.17 THEORETICAL CURVES FOR 1cm WIDE RIBBON

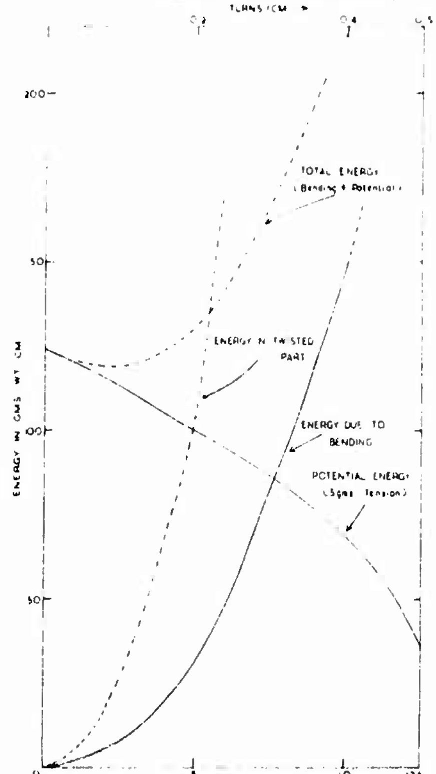


FIG 5.18 THEORETICAL CURVES FOR 1cm WIDE RIBBON

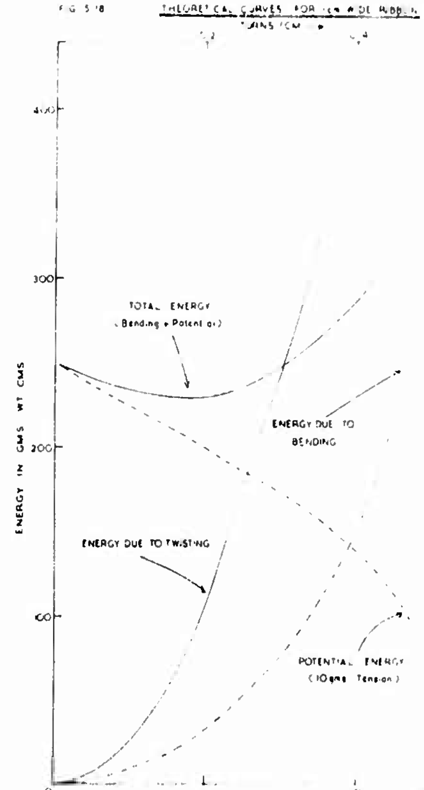


FIG 5.19 THEORETICAL CURVES FOR 0.5cm WIDE RIBBON

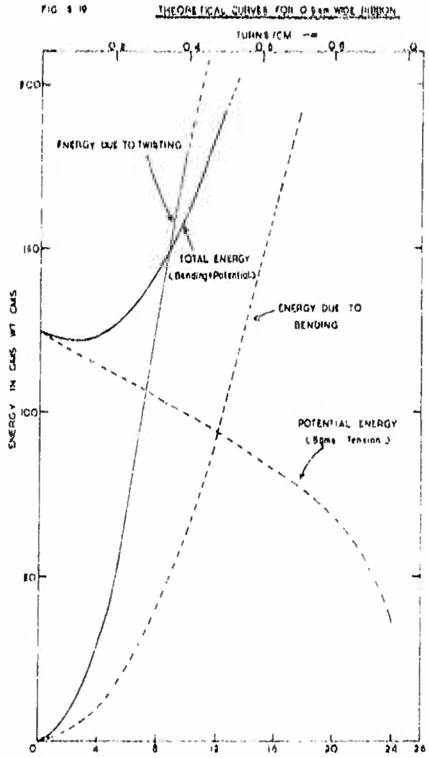
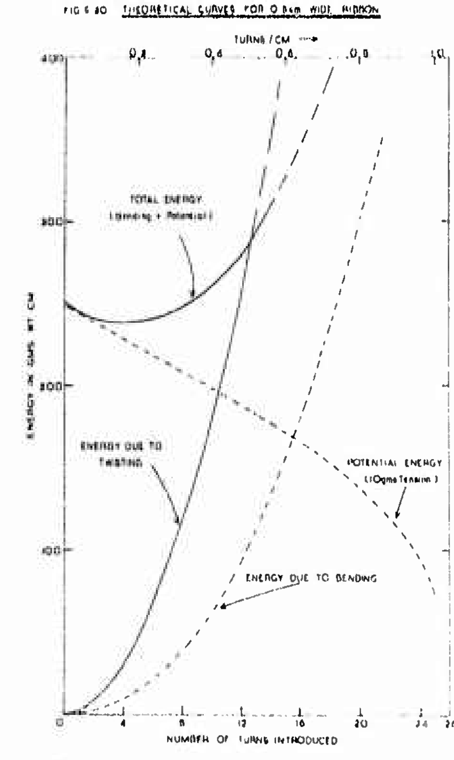


FIG 5.20 THEORETICAL CURVES FOR 0.5cm WIDE RIBBON



twist (a) $n_2 < \frac{\ell_2}{2b}$, (b) $n_2 = \frac{\ell_2}{2b}$, and (c) $n_2 > \frac{\ell_2}{2b}$. When the +ve sign solution is considered for the radius, it can be seen that the Potential energy decreases with gap for all the twists and also at any value of gap S the potential energy decreases with the twist. An exactly opposite behaviour is observed when the -ve sign solution is considered. This effect of decrease in Potential Energy (when the +ve sign solution is considered) can be explained by the fact that when a fixed length of ribbon is made to give a wrapped structure then at one turn the radius of the structure will be very large and hence will cause maximum contraction. When the number of turns are increased, the radius will decrease and hence a less contraction will occur. As the potential energy is directly proportional to the contraction a higher potential energy will be obtained at lower twists.

Again, it will be seen from the Fig. 5.15 that when $n_2 = \frac{\ell_2}{2b}$ the minimum potential energy occurs at zero gap whereas $n_2 < \frac{\ell_2}{2b}$, the minimum potential energy occurs when there is a gap in the wrapped structure. At values of $n_2 \lessgtr \frac{\ell_2}{2b}$, the minimum potential energy occurs when there is a gap in the wrapped structure. At values of $n_2 > \frac{\ell_2}{2b}$ the gap is not possible and the minimum potential energy occurs where there is a minimum overlap for a particular twist.

5.7 Total Energy in the Wrapped Form

The total energy stored in the wrapped form is the summation of energy due to bending and energy due to contraction in length during wrapping. As explained before the latter energy is stored as the potential energy of the system. Figure 5.16 shows that the total energy in the wrapped part decreases as the gap 'S' increases until a minimum value is reached and beyond that the energy increases once again. Thus when $n_2 = \frac{\ell_2}{2b}$ no structure with a gap is possible but a wrapped form with jammed structure is achieved. When $n_2 > \frac{\ell_2}{2b}$ only overlapped structure is possible but for values of $n_2 < \frac{\ell_2}{2b}$ structure with gap is possible. The energy values for all the above conditions show a minima in the overlapped region of the curves. This indicates that at a particular value of twist a structure with certain amount overlap as shown by the curves will be preferable.

5.8 Choice of Solutions

As observed before, the equations for the $(\text{radius})^2$ of the wrapped structure gives rise to two solutions, one when the positive sign is considered and second when the negative sign is considered. The solution with negative sign indicates that the radius increases as the twist is increased and also there is an increase in radius as the gap is increased (Fig. 5.8). Further it indicates that the total energy in the wrapped form decreases with the increase in twist (Fig. 5.16). In actual practice quite an opposite effect is observed, the increase in twist is always followed by reduction in diameter (when twisted under tension), and the energy also increases with twist. The positive sign solution satisfies these conditions of practical observations and hence will be used for our further theoretical developments.

5.9 Combination of Twisted and Wrapped Forms

When a strip of material is twisted by the application of an external force, it will attain a form for which minimum amount of work has to be done by the applied force. Hence in this form minimum amount of energy will be stored.

Referring to Fig. 5.16, it will be clear that for all the values of n_2 the minimum energy occurs in the region of small overlap, but since overlapping of edges is not possible in a ribbon of finite thickness, the minimum energy values will be estimated from the zero gap position. Thus, it will simplify upto certain extent the complicated calculations for the energy values at different twists.

In our estimation of energy from zero gap position certain amount of energy due to compression has been neglected which will be very small because the deformation due to compression is small and hence can be neglected.

Figures 5.17 to 5.20 show the energy values at various twists corresponding to the twisted form and wrapped form. It will be noted that the energy due to bending in the wrapped form is less than the energy due to torsion in the twisted form, indicating that the wrapped form will be favoured. However when potential energy in the wrapped form is added in, the situation changes. The potential energy is observed to have a finite value even at zero twist and is due to the weight by which it has been tensioned to prevent it contracting as when left free. At low twists, the twisted form has the lower energy and will thus be the stable configuration, but at high twists the wrapped form has the lower energy and hence will become stable. The intersection of the curves for energy in the twisted and the total energy in the wrapped part in Figs. 5.17 to 5.20 indicate the number of turns beyond which the wrapped form will appear.

It is now necessary to consider the possibility of the twisted and wrapped forms existing together. If a given number of turns n are introduced into the strip of length ℓ, then these may be divided in different proportions between different lengths in each form. However, stability will only be achieved when the torque in the twisted part is equal to the bending torque in the wrapped part.

When twisted under tension, there will be no torque acting on the twisted part of the structure due to this tension, but an additional torque will be acting on the wrapped part. If T_s

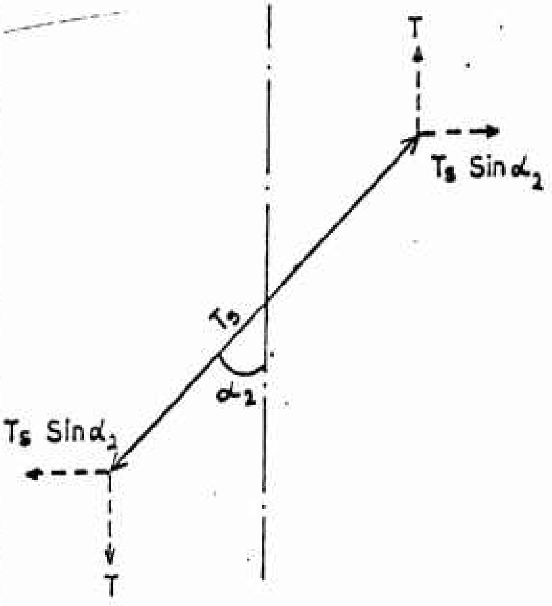


FIG. 5.21

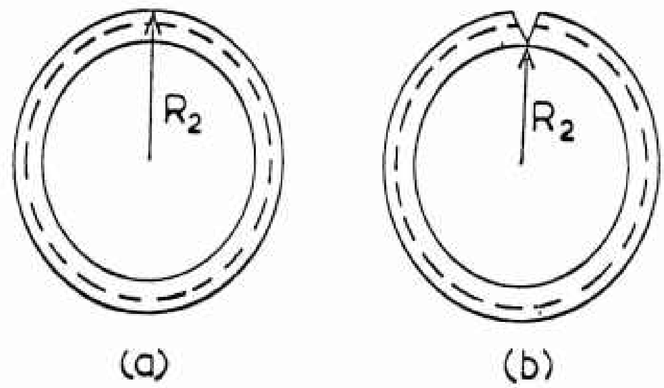
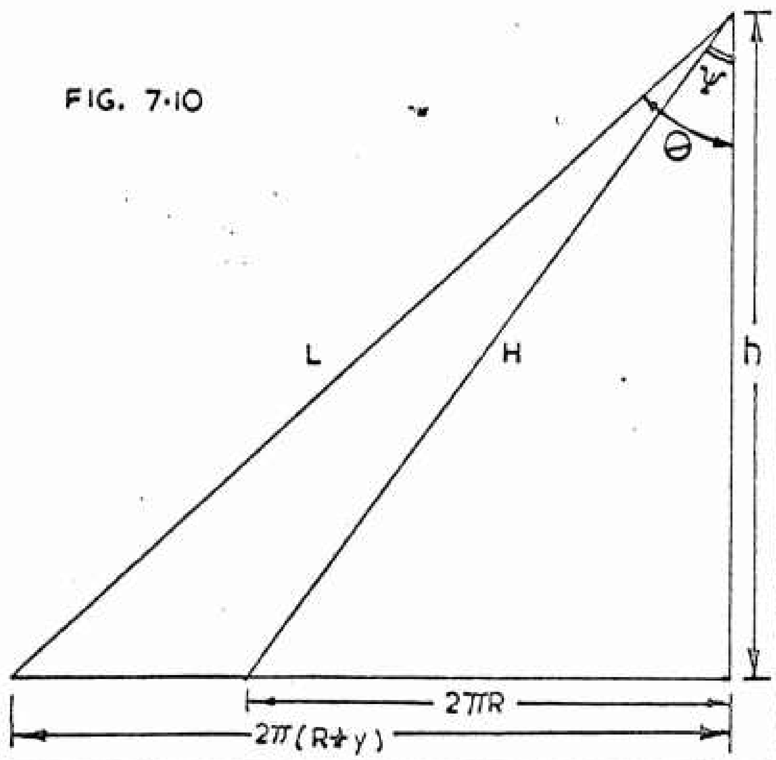


FIG. 5.32

FIG. 7.10



be the tension acting along the axis of the strip in the wrapped form (Fig. 5.21)

then the component $T_s \sin \alpha_2$ will be producing another bending couple acting on the ribbon causing it to bend more (α_2 being the helix angle). If T be the applied tension then

$$T_s \cos \alpha_2 = T \qquad \ldots\ldots \quad (5.27)$$

$$\therefore \quad T_s \sin \alpha_2 = T \tan \alpha_2 \qquad \ldots\ldots \quad (5.28)$$

If R_2 be the radius of the cylinder into which the strip has been bent then the moment of this bending couple will be $T.R_2 \tan \alpha_2$. Thus the total torque in the wrapped part = torque due to twist in the twisted part + torque due to tension. Hence torque for bending in the wrapped part due to torque in the twisted part will be obtained by subtracting the bending torque due to tension from the total torque in the wrapped part. In Figs. 5.22 to 5.24 the torque twist curves have been plotted. Thus if a given number of turns per unit length is selected for the twisted form, the number of turns per unit length for the wrapped form should be determined from the resultant torque twist curve as shown by dotted lines in Figures 5.22 to 5.24. The division of length and twist between the two forms then follows from the equations:

$$n = n_1 + n_2 \qquad \ldots\ldots \quad (5.29)$$

$$\ell = \ell_1 + \ell_2 \qquad \ldots\ldots \quad (5.30)$$

Thus from the above two equations

$$n = \frac{n_1}{\ell_1} \times \ell_1 + \frac{n_2}{\ell_2}(\ell - \ell_1) \qquad \ldots\ldots \quad (5.31)$$

It will be observed from Figures 5.22 and 5.23 that when the torque in the wrapped part has attained the maximum value (at 0.5 turns/cm. for a ribbon of 1 cm. width and 25 cm. length, and at 1.0 turn /cm. for a ribbon of 0.5 cm. width and 25 cm. length), any further increase in torque due to twisting will cause compression between the edges of the strip without altering the total number of turns in the strip. Only small amount of energy will be involved in the compression in our case because in our experimental observation twist to cause compression is not attained, hence has been neglected from our calculations at the

FIG. 5 22

TORQUE CURVES FOR 1cm WIDE RIBBON

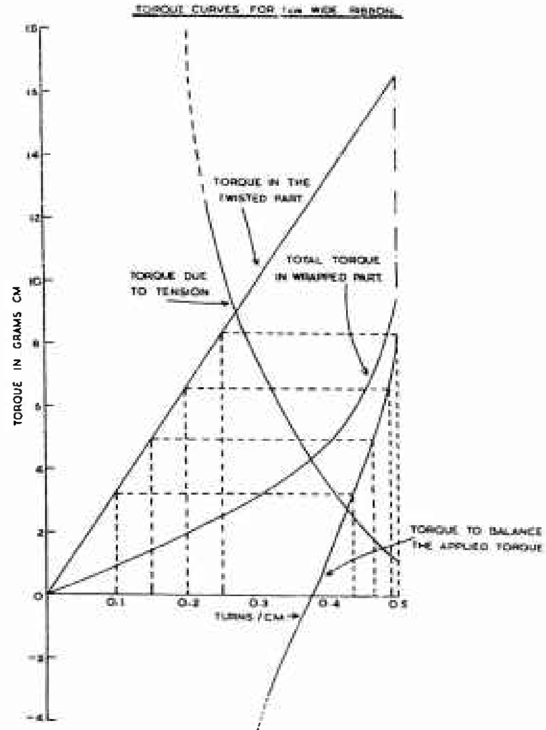


FIG 5 23

TORQUE CURVES FOR 0.5cm WIDE RIBBON

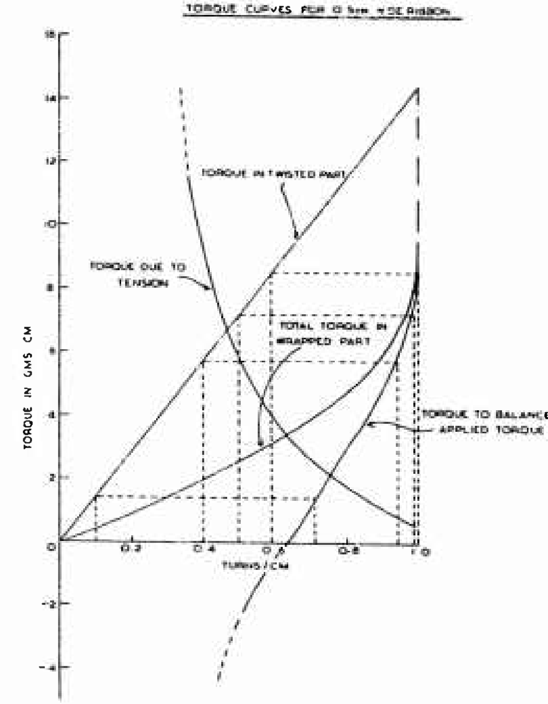


FIG 5 24

TOTAL ENERGY CURVES FOR 1cm WIDE RIBBON
(5gms Tension)

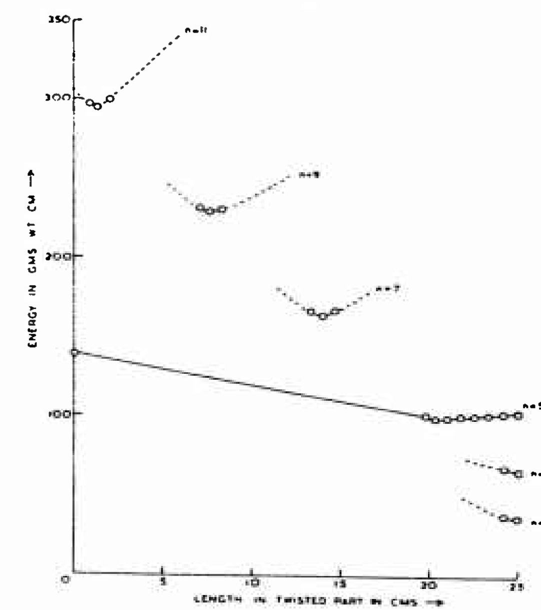


FIG 5.25

TOTAL ENERGY CURVES FOR 0.5cm WIDE RIBBON
(5gms Tension)

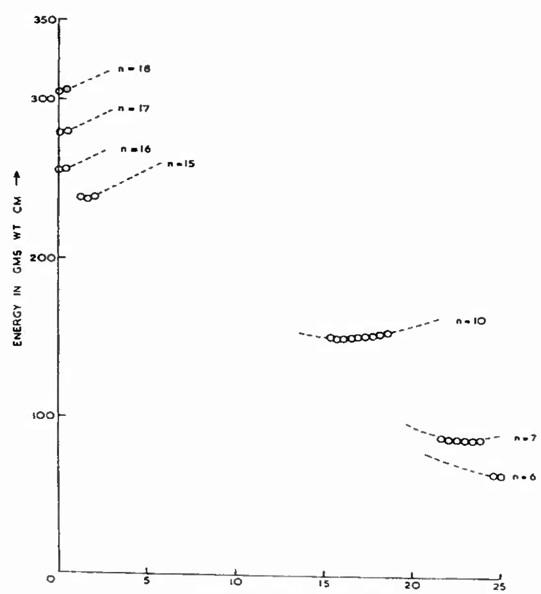


FIG 5 26

COMPARISON OF THEORETICAL & EXPERIMENTAL RESULTS FOR 1cm WIDE RIBBON
(5gms Tension)

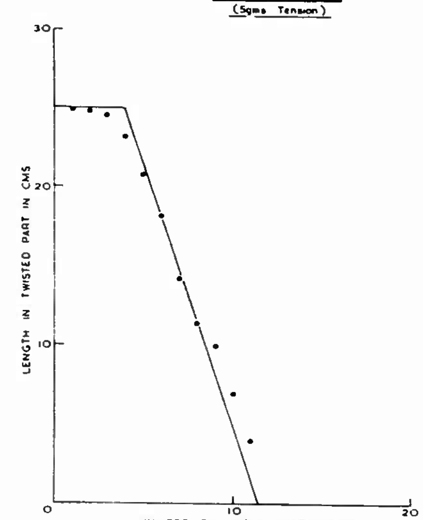
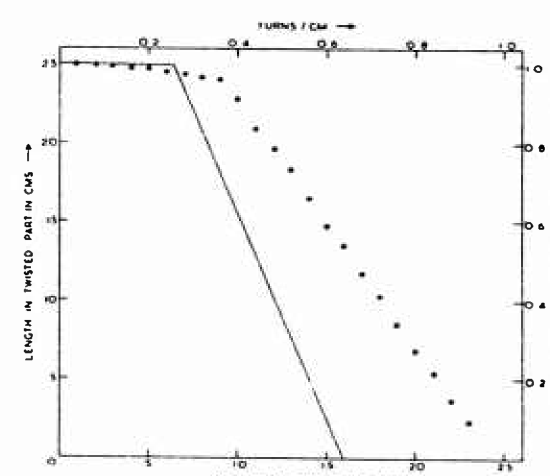


FIG 5 27

COMPARISON BETWEEN THEORETICAL & EXPERIMENTAL RESULTS
(When Corrections have been Applied)
(0.5cm Wide & 25cm Long Ribbon)

5gms Tension { • EXPERIMENTAL
— THEORETICAL



present stage.

The negative bending moment signifies that if a wrapped form is to be obtained with a few turns per cm., untwisting torque has to be applied to balance the torque due to the tension.

The total energy in the structure may be obtained by adding the appropriate combinations from the twisted and wrapped parts. An example of the calculations is given in table 5.1. The total energy in the structure at various twists for 1 cm. wide and 0.5 cm. wide ribbon has been calculated and a few energy values near the minimum energy position have been plotted against the length in twisted part in Figs. 5.24 and 5.25. The minima of these cruves represent the positions of stability and so the division between the wrapped and twisted form is established. It can be seen that at low twists the minimum energy occurs when there is a longer length in the twisted part but at higher twists, the minimum energy occurs when there is longer length in the wrapped part. This theoretical division is shown plotted against the number of turns in Figs. 5.26 and 5.27.

5.10 Comparison of Theoretical and Experimental Results

In Figs. 5.26 and 5.27 the theoretical and experimental results for the length of strip in the twisted part during twisting have been shown. It will be seen that the theoretical and experimental curves have got an excellent fitting for 1 cm.

wide ribbon but for 0.5 cm. wide ribbon the fit is not very good.

In Figs. 5.28 and 5.29 theoretical and experimental values for contraction in length dur_ng twisting have been plotted against the number of turns introduced. It will be seen that the initial portions of the theoretical curves show no contraction in length (according to our assumption) which agrees quite well with the experimental results. Near the end when nearly all the length of the strip has been wrapped a large difference in the contraction values between the theoretical and experimental results is obtained. This is due to the reason that as the strips are held in the flat jaws of the apparatus a certain length of strip can not wrap completely and thus giving a lesser contraction value than actual.

Figs. 5.30 and 5.31 compare the theoretical and experimental values of helix angles for 1 cm. wide and 0.5 cm. wide ribbons. For 1 cm. wide ribbon the theoretical and experimental curves agree quite well showing that the helix angle in the twisted part increases with twist until the wrapped form appears, after which the helix angle in twisted and wrapped part remains constant and any twist introduced changes the twisted form into wrapped form. The curves for 0.5 cm. wide ribbon show quite a contrary result. The theoretical curves show a decrease in the helix angle with twist in the twisted part after the wrapped form has appeared with an increase in the helix angle in the wrapped part.

FIG 5 29

COMPARISON OF THEORETICAL & EXPERIMENTAL CONTRACTION

(0 5cm Wide Ribbon & 25cms Long)

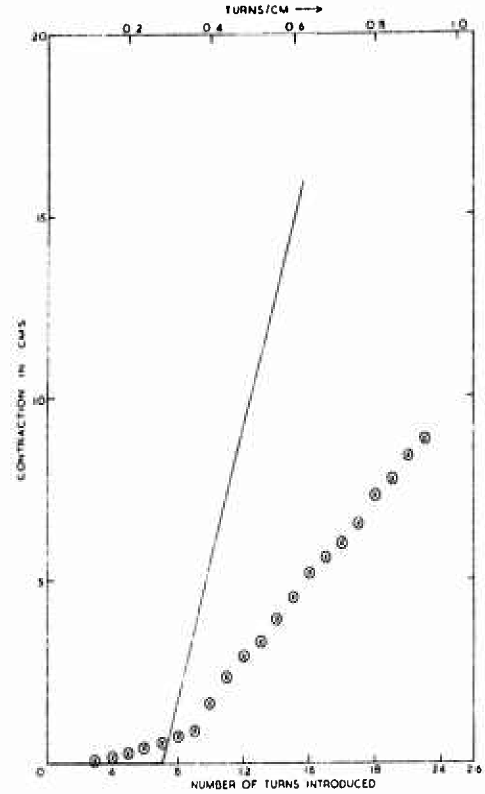


FIG 5 28

COMPARISON OF THEORETICAL & EXPERIMENTAL CONTRACTION IN LENGTH

(A Ribbon of 1cm Width & 25cms Length)

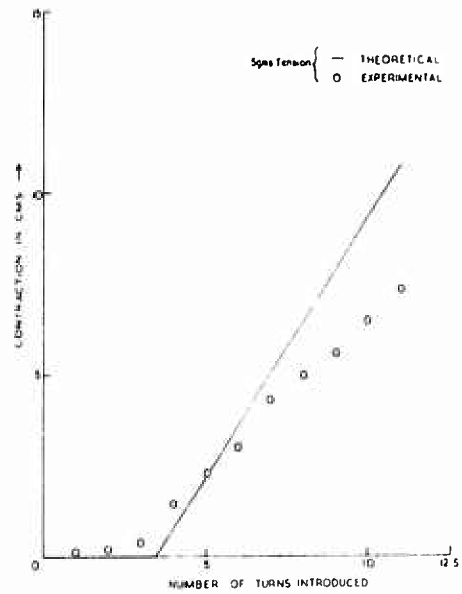


FIG 5 30

COMPARISON OF THEORETICAL & EXPERIMENTAL CURVES

HELIX ANGLE FOR 1cm WIDE RIBBON

(5gms Tension)

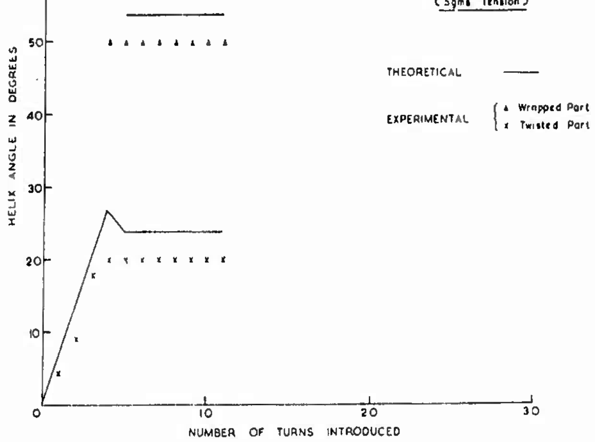
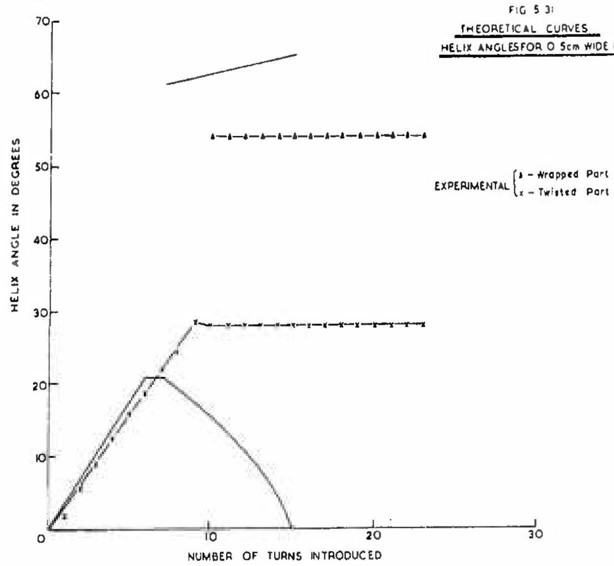


FIG 5 31

THEORETICAL CURVES

HELIX ANGLES FOR 0 5cm WIDE RIBBON



The experimental results show that the angle in both the parts remains constant.

5.11 Discussion of Results

The above comparison of theoretical and experimental results shows that the agreement between them is excellent in case of 1 cm. wide ribbon but the fit is poor in case of 0.5 cm. wide ribbon. Nevertheless the basic agreements can be explained as follows:

(a) When a ribbon is twisted the number of turns introduced goes to form the twisted structure and the helix angle of the structure increases slowly.

(b) The contraction in length during twisting is very small until the wrapped form appears when the contraction increases rapidly.

Any disagreement between the theoretical and experimental results may be due to the following causes:

(i) Neglect of the effect of radius at which wrapped structure jams.

In our calculations for the radius of wrapped structure at zero gap, the structure was assumed to have been jammed at the middle of the thickness of strip (Fig. 5.32a). This is nearly true in case of very wide ribbons (1 cm. wide, where $\frac{b}{d} = 10$ or over), but in case of narrow ribbons (0.5 cm. wide where

$\frac{b}{d} = 5$) the jamming of the structure takes place in the inner layer as shown in Fig. 5.32(b). This will in effect reduce the experimental values of

contraction in the wrapped part and hence a smaller energy than the calculated value will be stored in the wrapped part. Since the torque in the wrapped part is inversely proportional to the radius^2 (R_2^2), the experimental torque values will be higher than the calculated values and also the energy stored in the system will be larger than the theoretical values.

This effect is greatly magnified in the case of narrower ribbons and hence the disagreement between the theoretical and experimental values is increased.

(ii) Neglect of the energy relations at the boundary between the twisted and wrapped forms.

Going back to the geometry of two forms it can be observed that if a line was drawn along the width of the ribbon perpendicular to its axis, then in twisted form this line will remain perpendicular to the axis of this form but same line will be inclined at an angle to the axis of the wrapped form. This is quite true in case of 1 cm. wide ribbon, but 0.5 cm. wide and narrower ribbons this line becomes inclined to the axis of twist when wrapped form is approaching. This will include certain amount of shear energy in the system.

(iii) General assumptions of the small strain elastic theory in a problem involving large strains.

Although corrections for large strain have been applied the fundamental equations for the elastic theory were arrived at by considering small strain.

(iv) Experimental error, though the only possible source of appreciative error here is non-uniformity in the rubber strip leading to inappropriate values of the moduli used in calculations.

From above discussions it will be clear that theoretically a number of forms can be available. The structure might be twisted form or a wrapped form with or without gap. The form which will have minimum energy will be obtained in practical cases.

CHAPTER 6

STUDY OF THE TWISTING OF FILAMENT BUNDLES

6.1 Introduction

The experimental and theoretical studies have shown with fair agreement how a flat rubber strip can twist in two quite distinct forms. In this chapter the work has been extended to see how similar ideas apply in the twisting of filament bundles. Three ideal forms of twisted structures have to be investigated:

(a) Twisted cylindrical bundles

(b) Twisted flat ribbon

(c) Wrapped flat ribbon.

Once again, for convenience, rubber filaments ('Shirelastic' threads - rubber filaments covered outside with cotton thread) of circular cross-section were chosen for the study.

The study of the twisted structure was divided into two sections.

(1) An assembly of filaments in a ribbon form and observed under three different types of twisting:

(i) Static Twisting

(ii) Twisting at Constant Tension

(iii) Continuous Twisting.

(2) A cylindrical assembly of filaments and twisted on the continuous twisting type machine.

6.2 Description of the Apparatus Used:

6.2.1 Twisting of A Ribbon of Filaments

(i) For making observations during static twisting the ribbon was twisted on a twist tester.

(ii) Twisting at constant tension - A fixed length of ribbon was held between a jaw which could be rotated and the jaw of a trolley sliding on rails. (A description of this apparatus has been given in Chapter 4).

(iii) Continuous twisting of ribbon - A length of filaments was delivered by this machine and twist was imparted at the same time. The length was taken up by the movement of twisting unit backwards as twist was imparted. The twisting unit was made the controlling unit for delivering the length of filaments.

The twisting unit of the apparatus is the same as that used for twisting solder wire (Chapter 3). A feed system was attached to this twisting unit to deliver filaments as twist was imparted. The delivery roller assembly used for this purpose was the same as the delivery roller assembly described in Section 3.3.1(c). Fig. 6.1 shows the full assembly of the twisting unit.

In order to get a purely ribbon form of twisting the

FIG 6.2

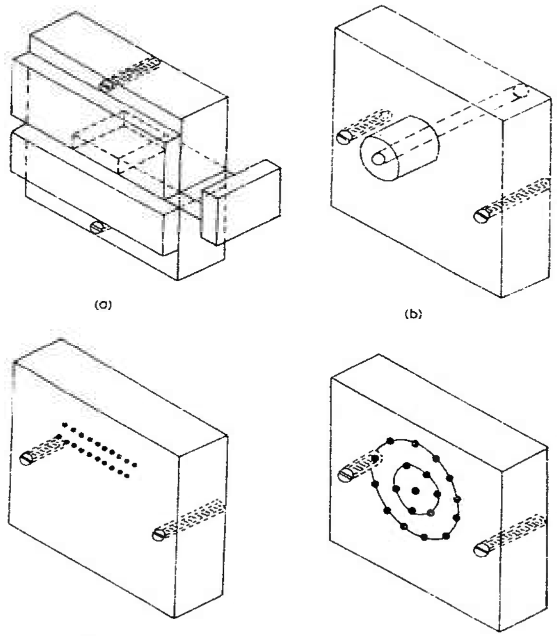


FIG 6.3

STATIC TWISTING OF ONE LAYER OF TEN THREADS

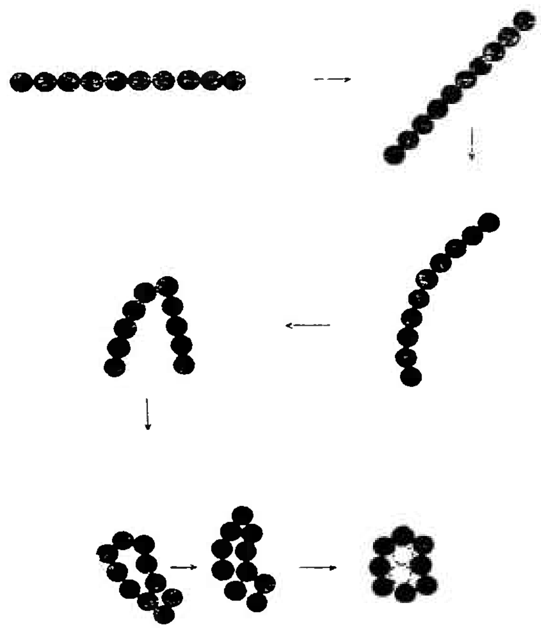


FIG 6.4

TWISTING OF ONE LAYER OF TEN THREADS AT CONSTANT TENSION

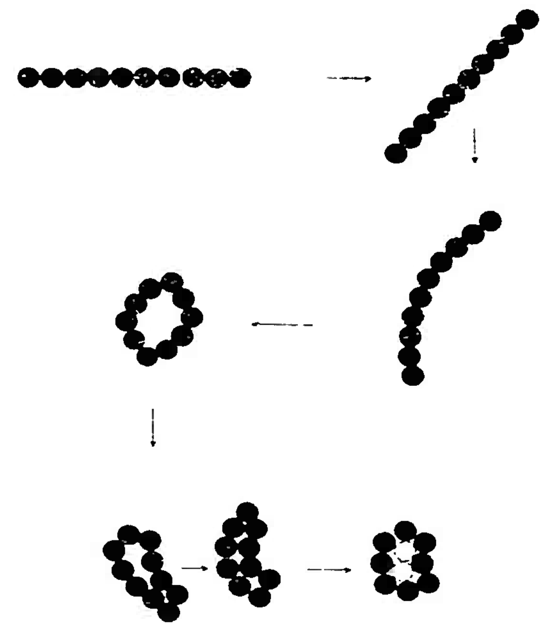


FIG 6.5

CONTINUOUS TWISTING OF ONE LAYER OF TEN THREADS

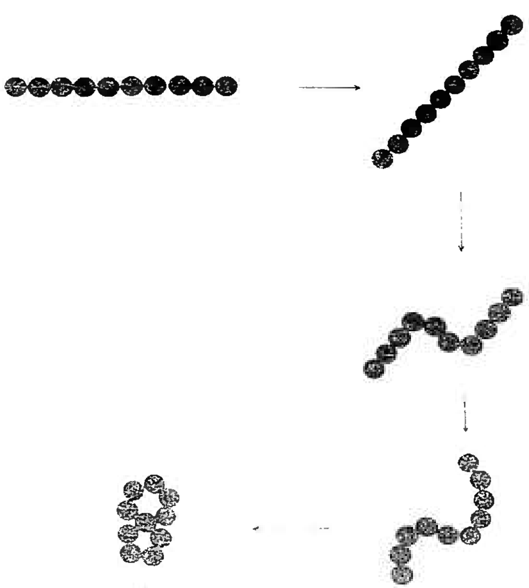


FIG 6.6

STATIC TWISTING OF TWO LAYERS OF TEN THREADS

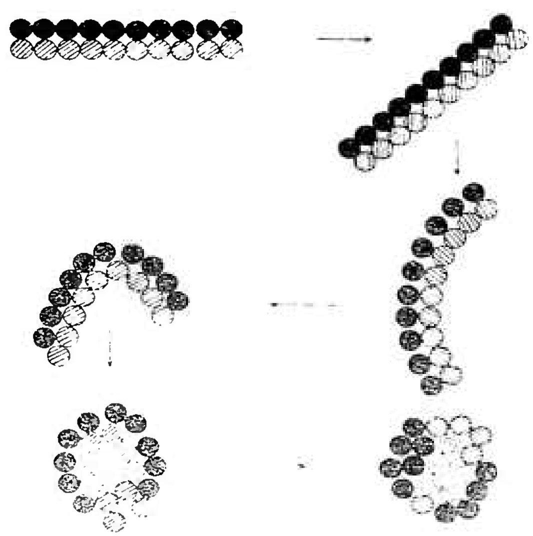
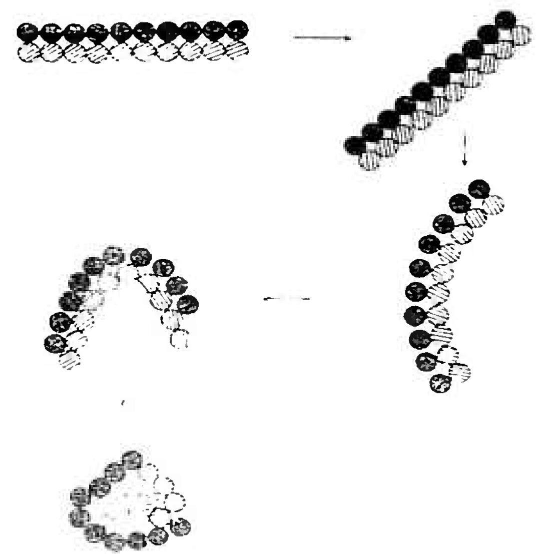


FIG 6.7

TWISTING OF TWO LAYERS OF TEN THREADS AT CONSTANT TENSION



filaments were passed through a slot made on a $^1/_8$" thick Perspex plate. A T-shaped Perspex block of the same size as the slot on the Perspex plate was pushed into the slot until the width of the slot was the same as the width of the ribbon of filaments used. Thus the width of the slot could be adjusted according to the width of the ribbon of filaments. Two edges of Perspex plates were kept pressed against the ribbon of filaments on either face of the main Perspex plate with slot, to ensure that the filaments are fed in a ribbon form. These auxiliary Perspex plates on either face of the main Perspex plate with slot could be moved apart and can be adjusted to feed different thicknesses of ribbon. Care was taken to mount this assembly on the twisting machine in front of the delivery rollers such that the slot, the twisting jaw and the nip of the delivery rollers were in the same horizontal plane.

One end of a flexible shaft was attached to the bottom shaft of the twisting unit and the other end was connected to the bottom delivery rollers. Thus the twisting unit would control the delivery of the filaments. As the pitch circle diameter of the pinion on the rack was one inch and the diameter of the delivery rollers was one inch, one complete rotation of the bottom shaft would move the twisting carriage by 3.14 inches on the rack and at the same time 3.14 inches of ribbon were fed by the delivery rollers. By using a varied combination of gear wheels on the twisting head the turns per unit length introduced during twisting could be altered.

A single layer of ten filaments and two layers of ten filaments were twisted and their structure studied. When making a ribbon, the filaments were laid side by side parallel to each other and were passed over a smooth glass rod and they were then individually tensioned. The front end of this ribbon was then cello-taped and was passed through the delivery rollers, the ribbon forming plate on to the twisting jaw. When twisting two layers of filaments, one layer was made by using coloured shire-lastic and the other was kept white. This helped in observing how different layers of filaments were behaving during twisting.

6.2.2 Twisting of filaments arranged cylindrically

To study the cylindrically twisted structure a different attachment had to be made to feed the filaments as a cylindrical bundle to the twisting jaw after they have emerged out of the delivery rollers. A small hole was drilled into a Perspex rod such that the diameter of the hole would be the same as the diameter of the circle when the filaments were arranged in a cylindrical form. This attachment was then mounted on the twisting apparatus in front of the delivery rollers instead of the ribbon forming unit. A fine hole was drilled through the Perspex rod in a direction perpendicular to its length and a needle was passed through this hole. This prevented the twist passing through on to the other end of this cylindrical feeding attachment. Fig. 6.2(b). This needle also served the purpose of preventing the two layers of different coloured filaments from mixing together until they came up to the point of twist formation. Thus all the white elastic filaments were passed through from one half side of this diametrical wire and the blue filaments from the other half.

The cylindrically twisted structure was also studied by feeding the filaments in a circular array by passing them through a spacer. The spacer was made by drilling holes on a Perspex plate. The holes were drilled on concentric circles of radius r, 2r, etc. such that there was one hole in the centre of the circle, six holes equally spaced on the first circle, twelve holes equally spaced on the second circle and so on. A diagram of this attachment is shown in Fig. 6.2(d). This attachment was mounted on the twisting part of the apparatus in front of the delivery rollers. This method of cylindrical feed differs from the previous method that the filaments are not fed in a bundle.

6.3 Observations

The twisting process was carried very slowly and the behaviour of different coloured filaments at each stage of twisting were carefully noted. The number of filaments of different colours appearing on the surface were counted and their positions located. Thus a cross-section of yarn was drawn. Although this cross-section may not be exactly the same as the cross-section of the yarn if it were imbedded and then sectioned, yet it will give us nearly a very true picture of the structure of yarns. The filaments which went into the core of the yarn structure and their positions could not exactly be located, were drawn with dotted lines. The two different shades in the figures distinguish the colour of filaments used.

6.3.1 Ribbon Twisting

(A) Single Layer (i) Static Twisting It will be observed that the twisted ribbon form stays for about a quarter of a turn and after that the edges of the ribbon begin to curve. When about half a turn has been introduced, the ribbon bends from the middle into a $\wedge$ -shaped structure. On introducing further twist the gap between the two edges begins to close, and when one full turn has been introduced a structure which has got a hollow in the centre is obtained but the structure is not circular. It is an elliptical sort of structure with one end of the major axis flattened. On twisting this structure further, it twists as two layers of filaments and gives rise to twisted ribbon structure again with a narrower ribbon width and finally we get a structure which is still not circular but is elliptical with very little difference in major and minor axes. Fig. 6.3.

(ii) Twisting at constant tension. The first three stages of structure observed in this case are similar to the structures in the previous case but after that instead of the ribbon breaking from the middle into a $\wedge$ -shaped structure, the edges of ribbon cruve more until finally at one complete turn we get a structure with hollow centre but the yarn does not appear cylindrical from outside. If the twisting is allowed to proceed on, this structure collapses into a flat ribbon having two layers and the twisting of a flat ribbon continues once again. Fig. 6.4.

(iii) Continuous twisting. In this case also twisted ribbon structure is obtained up to a quarter of a turn but further twisting leads the flat ribbon to fold into an N-shaped structure. Beyond half a turn until one turn the long edges of the N-shaped structure keep on curving and at one full turn we get a flat structure with one filament in the centre and nine filaments forming the outer boundary of the yarn. Obviously a certain amount of gap at the centre is present as well. When higher turns per inch are introduced in this twisting, the structure mentioned above flattens out and starts twisting as a narrower but thicker ribbon until finally a cylindrical appearance of yarn is obtained. Fig. 6.5.

(B) Two Layers (i) Static Twisting The twisted ribbon structure was obtained for a quarter of a twist and beyond that the edges of the ribbon began to curve and this curvature went on increasing until at one full turn a hollow centred structure was obtained. It will be observed that the structure is not exactly circular and all the blue coloured filaments are not completely covered by the white filaments. The explanation is quite simple, the space inside the hollow cylinder formed by the white filaments is not sufficient to hold all the blue filaments which are the same in number as the white ones. Hence certain number of coloured filaments were visible on the surface of the twisted yarn. On further twisting this hollow structure, it assumes a closely packed structure, but it does not attain the theoretical structure of hexagonal packing. Fig. 6.6.

(ii) Twisting at Constant Tension. The structures at different stages of twisting are similar to the previous case but when one complete turn has been introduced, the structure seems to become polygonal with loose packing in the centre. Fig. 6.7.

(iii) Continuous Twisting. In this experiment the yarn was twisted by feeding the filaments in various ways and also gripping them in the jaw in different ways. The effect of different forms of feeding and gripping in jaws were examined.

(a) The filaments were fed through a slot of rectangular shape and gripped at the jaws in a similar fashion by laying the filaments parallel and close to each other.(At 1.91 turns/inch). The twisted ribbon form is stable until a quarter of a turn, and on further twisting the ribbon breaks itself into an N-shaped structure. The filaments keep re-arranging their position until at about three quarters of a turn it will be observed that the white filaments are all on the circumference of the yarn while the coloured filaments are trying to acquire the position in the centre of the yarn. On giving a full turn a close packed structure but non cylindrical in shape is obtained. Fig. 6.8.

(At 3.5 turns/inch). Similar stages of structures are obtained when higher turns/inch are introduced, but when a full turn has been introduced a close packed, nearly circular structure is obtained. It will be observed that due to high twist distortion in the shape of the filaments has taken place. Fig. 6.9.

(b) In this study the filaments were fed through a spacer and were held in the jaw in a ribbon form by laying the filaments parallel and close to each other. A close packed nearly cylindrical structure is obtained. The arrangement of

FIG 6.14

CONTINUOUS TWISTING OF TWO LAYERS OF TEN FILAMENTS PASSED THROUGH A SPACER CYLINDRICALLY AND HELD IN THE JAW IN A CYLINDRICAL BUNCH

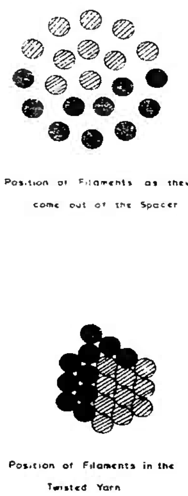


FIG 6.15

CONTINUOUS TWISTING OF FILAMENTS DELIVERED CYLINDRICALLY BY PASSING THROUGH A SPACER & HELD IN THE JAW IN A SIMILAR FASHION

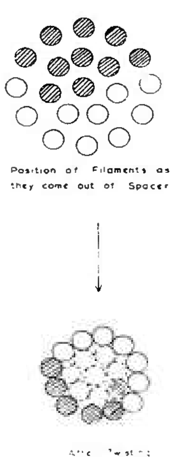


FIG 6.12

CONSTANT TWISTING OF TWO LAYERS OF TEN FILAMENTS PASSED THROUGH SPACER (in Ribbon Form) AND HELD IN THE JAW SPACED THE SAME WAY

(3.11 t.p.i.)

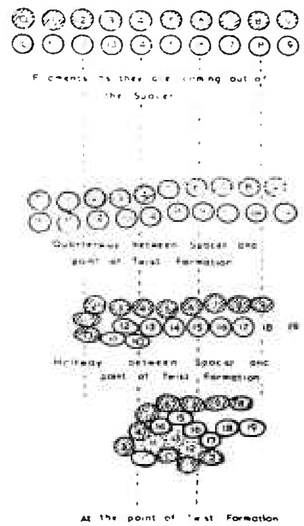


FIG 6.13

CONTINUOUS TWISTING OF TWO LAYERS OF TEN THREADS FED IN CIRCULAR FORM

FIG 6.10

CONTINUOUS TWISTING OF TWO LAYERS OF TEN FILAMENTS FED THROUGH A SPACER IN RIBBON FORM AND THE ENDS ARE HELD IN THE JAW BY LAYING THEM CLOSELY PARALLEL TO EACH OTHER

FIG 6.8

CONTINUOUS TWISTING OF TWO LAYERS OF TEN THREADS

(1.9 t.p.i.)

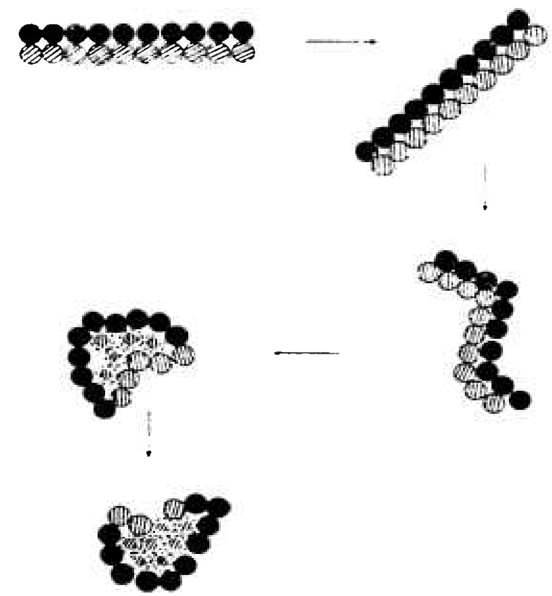
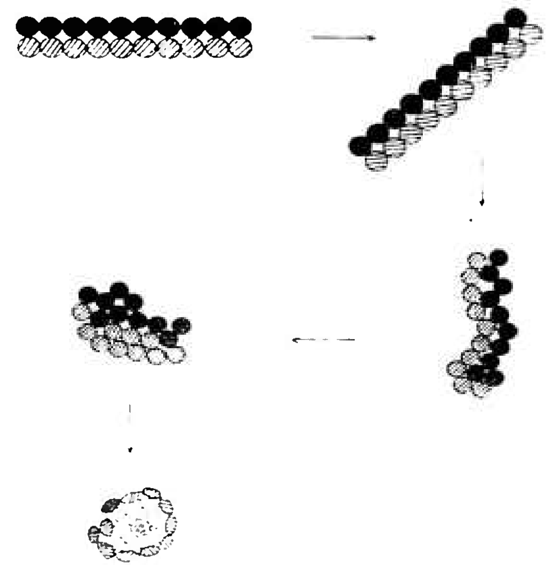


FIG 6.9

CONTINUOUS TWISTING OF TWO LAYERS OF TEN THREADS

(Starting Length 3") (2.5 t.p.i.)



filaments inside the yarn has been shown in Fig. 6.10

(c) In the third experiment the filaments were evenly spaced by feeding the filaments through a spacer and were also gripped in the jaw evenly spaced the similar way. When this assembly of filaments was twisted it was observed that the twisted ribbon form was obtained for about quarter of a turn and then on further twisting an N-shaped structure was obtained. On twisting further it was observed that the ribbon was twisting in a way such that one edge of the ribbon was overlapping over the other. This overlapping of the edges of the ribbon is of the similar nature as explained in the paper model of ribbon twisting in Fig. 5.3(b). A photograph showing the overlapping of edges of the ribbon in this particular case can be seen in Fig. 6.11. To demonstrate, stage by stage, how the overlapping is taking place, filament positions in the cross-section of the yarn have been drawn. The cross-sections of the yarn have been taken at various positions between the nip of the delivery rollers to the point of yarn formation. In the cross-section the filaments were numbered to indicate their position at different stages and the dotted lines are the reference lines with respect to the original position of the filaments. Fig. 6.12.

Conclusions

From above studies it is clear that when a ribbon of filaments, where each individual filament is free to rearrange its position, is twisted, it gives rise to two forms of structure (a) the twisted form and (b) the wrapped form. The twisted form seems to be very unstable and could exist only at extremely low twists. The wrapped form of structure which is observed during the process of twisting does not give rise to a hollow centred structure, as it is obtained in case of twisting of uniform strips, instead, the wrapped structure which is initially formed with a hollow centre collapses and gives rise to a uniformly packed yarn cross-section. In certain cases overlapping of filaments at one edge of the ribbon over the filaments at the other edge has been observed. This overlapping of filaments may tend to cluster the filaments in core of the final yarn.

Examining the cross-sections of the yarns, specially those made of two coloured layers, it will be seen that the statically twisted yarns do not give any idea about the wrapped structure. The coloured and white filaments are uniformly distributed in the cross-section. This might be due to the fact that a long length of yarn is twisted in static twisting and the filaments re-arrange due to high tension developed in the extreme filaments.

The cross-sectionsof yarns twisted on a continuous twisting type machine indicate the presence of wrapped form of structure. In the yarns twisted at 1.9 t.p.i. the cross-section will show wrapped form but the coloured filaments are not completely surrounded by the white filaments. At 3.5 t.p.i. it can be seen that nearly complete surrounding of coloured filaments by white ones have taken place and is the nearest approach to the wrapped structure,

6.3.2 Cylindrically Twisted Yarns

All the cylindrically twisted yarns were twisted on continuous twisting type of apparatus, but different methods of cylindrical feed were used.

(a) The filaments were fed in a closely packed cylindrical bunch and gripped in the jaw in a similar fashion. Cross-section of yarn is shown in Fig. 6.13.

(b) The filaments were fed through a spacer and were held in the jaw in a cylindrical bunch. The coloured filaments were passed from one half of the spacer and the white filaments from the other half. Cross-section of yarn is shown in Fig. 6.14.

(c) The filaments were fed through a spacer and were held in the jaw similarly spaced by passing them through a rubber spacer. Fig. 6.15 shows the yarn cross-section twisted by this method.

Conclusions

Observations on the cylindrical twisting indicate that there is a uniform packing of the filaments in the yarn. The yarn cross-section has the resemblance to hexagonal packing but the two layers of different coloured filaments remain as two separate layers even after twisting.

CHAPTER 7

CONSEQUENCES OF THE IDEAL FORMS

7.1 Introduction

The theoretical and experimental studies made up till now suggest that there are three possible ideal forms of yarn structure. They are

(a) Twisted cylindrical bundles.

(b) Twisted flat ribbon.

(c) Wrapped flat ribbon.

It will be argued in this chapter how far these three idealised structures could occur in practice, and their geometrical theories will be developed.

7.2 Twisted Cylindrical Structure

The idealised cylindrical structure can be considered as having been formed by twisting a cylindrical bundle of parallel filaments arranged co-axially. Figure 7.1(a) shows an assembly of parallel filaments each of length ℓ, such that their cross-section will be a circle of radius R. The structure of this assembly of filaments can be completely explained by the length ℓ and radius R. The position of any filament in this cylinder will be given by its radial distance from the centre of the cylinder.

Figure 7.1(b) shows the structure after certain number of turns have been imparted to the cylindrical assembly in Fig. 7.1(a). The theoretical model assumes that the filaments in this single yarn correspond to a set of co-axial helices, each having the same twist (number of turns in a given length of yarn), with the further assumption that the filaments are completely in extensible.

In such an idealised structure if the ends are not gripped the filaments constituting it will fall apart as there is neither interlocking of filaments nor any cohesive force present to give a stable structure.

Further considerations show that the assumptions of the theoretical model are mutually inconsistent. In the untwisted yarn the filaments are of equal length; in the final twisted state the length of a helix increases from the middle to the outside of the yarn. The final state is therefore incompatible with the assumptions that the filaments are inextensible. If the filaments do not stretch significantly, it follows that the final state can not correspond to a set of co-axial helices.

During twisting the fibres of a yarn must be in varying states of tension depending on the position they occupy, those lying on the surface being at a higher tension than those near the core. Therefore, it can be imagined that, on removal of external restraint represented by the spinning tension, the most likely condition of the resulting yarn would be one in which the

FIG 7.1

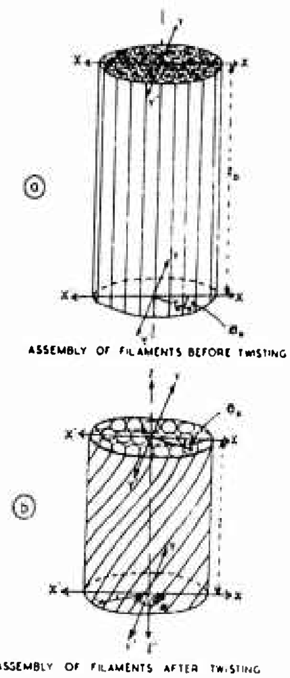


FIG 7.2

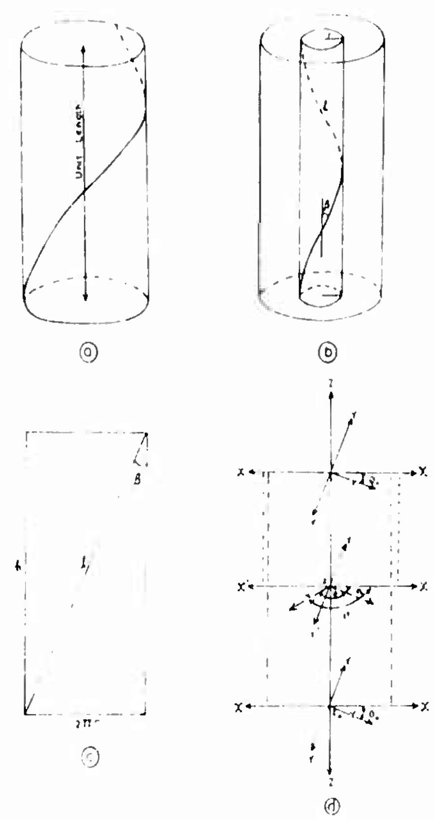


FIG 7.3

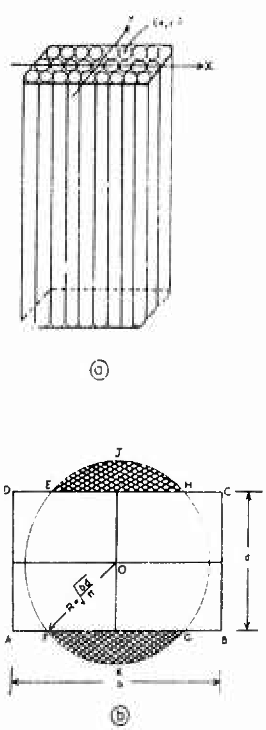


FIG 7.5

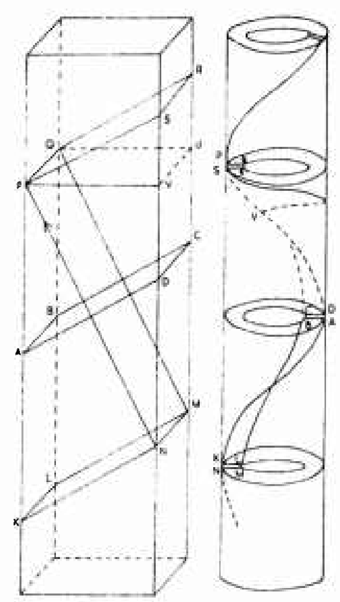


FIG 7.6

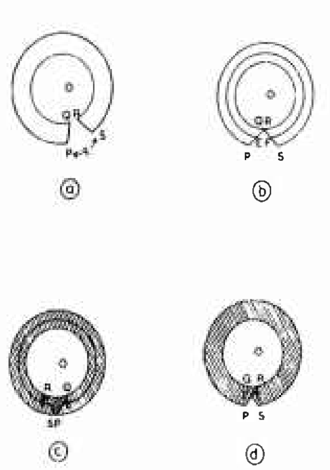
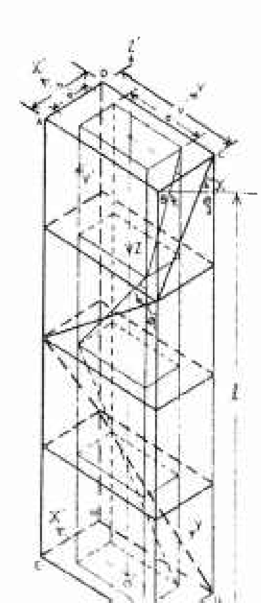


FIG 7.7



surface fibres are at some slight tension and the core fibres are buckled. This is in fact the kind of structure that is obtained if a bundle of filaments are twisted together over a long length, for example in a twist tester.

In continuous spinning, however, the situation is different. Confining our attention for the moment to a number of filaments being presented by a pair of rollers to the twisting action of the ring spindle, then at any instant - as the filaments are emerging from the delivery rollers and immediately before they take up their final position in the yarn - those which have been going to the outside will be taut, while those going to the inside will be slack or even buckled. It is natural to suppose, therefore, that if they can, the tight filaments will attempt to ease themselves of their strain by trying to acquire the more comfortable core position, while the slack ones will be gradually pushed to the surface. Thus moving along the yarn it would be found that the filaments are constantly changing places, from the outside of the yarn to the inside and back again indefinitely.

If this process was perfectly adapted to the changing conditions of tension, that is if the reactions of the filaments in the way described were instantaneous, the result would be yarn in every short element of which the component filaments are all of same length. Such a condition, however, is incompatible with the requirement that the pitch of the spiral should be constant. Furthermore, in practice the process of migration must suffer from a certain time lag, because some force is required to move a given filament from one position to another. Hence some degree of inequality of stress must be built up before necessary forces are generated. It may be concluded therefore that migration of this kind is to be expected, but at a rate slower than would give equal lengths of all filaments in any given length of yarn. Filament migration of this nature has been observed by Morton and Merchant.

From the above arguments it will be clear that if an ideal helical structure is assumed, we will be led to a final condition in which the fibres must be strained or the structure distorted if all the filaments are to extend over the same length of yarn. If this happens, then any simple geometrical theory of contraction must break down. The only condition under which a geometric theory can be applied is one in which there is an effective averaging in the actual yarn caused by the helical path of a filament varying in radius along the yarn.

To give equal weights to the fibres at all positions of the yarn it may be assumed that the deficiency in the length of filaments when they are near the surface is made up by the excess occurring when they are near the centre. Thus virtually there is a redistribution of excess length of filaments in the twisted yarn.

When a straight bundle of filaments is twisted under the condition that the filaments do not change their length due to straining, then the final length of the yarn produced will be shorter than the length of the filaments. This is due to the fact that the length of the helix the filament has to follow between two points on the yarn is larger than the straight length between these two points. The difference of the two lengths expressed as the percentage of the original length of the filaments gives the percentage retraction.

7.2.1 Geometry

Since the length of helix increases from the axis of the yarn outwards this implies that the outermost filaments have a greater length than those near the axis. Therefore to determine mathematically the contraction or the retraction an average value for the filament contraction or retraction between the centre and outside of the yarn has to be worked out.

Let $\bar{\ell}$ be the mean length of filaments in a length of twisted yarn h, and if it is assumed that the yarn helices migrate to and fro from the yarn axis, the length of the untwisted yarn will be equal to $\bar{\ell}$ and the retraction, (Fig. 7.2) (16)

$$Ry = \frac{\bar{\ell} - h}{\bar{\ell}} = \frac{1 - \cos\delta}{1 + \cos\delta} \qquad \ldots\ldots \quad (7.1)$$

Contraction factor $C = \frac{1}{2}(1 + \sec\delta)$ where δ = value of β, the angle of twist, at the yarn surface

$$\text{and} \quad \frac{2\pi r}{h} = \tan \beta. \qquad \ldots\ldots \quad (7.2)$$

In a cylindrical assembly of filaments the position of the filament can be defined in the following manner - Fig. 7.2(d)

Distance from the yarn centre $= r_o$

length along yarn $= z_o$

angle to a given direction in plane perpendicular to yarn axis $= \phi_o$

If r be the distance of the same filament from the yarn centre after twisting and z be its length along the yarn then if the yarn volume is assumed to be constant before and after twisting -

$$\pi r_o^2 z_o = \pi r^2 z \qquad \ldots\ldots \quad (7.3)$$

$$\therefore \quad r = \left(\frac{z_o}{z}\right)^{1/2} . \, r_o$$

but we know that

$$\frac{z_o}{z} = \text{contraction } C$$

$$\therefore \quad r = C^{1/2} \, r_o \qquad \ldots\ldots \quad (7.4)$$

Again from the eqn. (7.3)

$$\pi r_o^2 z_o = \pi r^2 z$$

$$\text{or} \quad \pi r_o^2 z_o = \pi C r_o^2 Z$$

$$\therefore \quad z = \left(\frac{1}{C}\right) z_o \qquad \ldots\ldots \quad (7.5)$$

If τ be the twist in turns/unit length (along the yarn) introduced as a result of which the reference filament has rotated through an angle ϕ from the axis of x as in Fig. 7.2(d) then

$$\phi = \phi_0 + \theta$$

$$= \phi_0 + (2\pi\tau)z_0 \qquad \ldots\ldots \quad (7.6)$$

The helix angle which a filament makes with the yarn axis is given by

$\tan\alpha = 2\pi R\tau$ where R is the value of r_0 at the surface of the yarn.

7.3 Twisted Flat Ribbon

The idealised structure in this case can be assumed as having been formed by twisting an assembly of parallel filaments of length 'ℓ' arranged so that the cross section of such an assembly will be a rectangle of width 'b' and thickness 'd'. The position of any filament in such a rectangular array of filaments will be denoted by the co-ordinates (x,y) with respect to co-ordinates the origin of which coincides with the centre of the rectangle and the axes of which are parallel to its sides. (Fig. 7.3(a)).

On twisting such an assembly in a way such that one end of the assembly remains fixed and the other end is rotated with respect to this fixed end, either in the clockwise or anticlockwise direction, the idealised structure of the twisted ribbon form will be obtained. Thus the central axis of the rectangular assembly is maintained as the axis of rotation. For further discussions this axis of rotation will be referred to as the core of the yarn.

The idealised twisted ribbon structure can hence be typified by the following characteristics.

(1) The yarn is uniform along its length and has a rectangular cross-section.

(2) The yarn is built up of a number of superimposed concentric rectangles such that centre of these rectangles lies on the central axis of yarn in the x-sectional view.

(3) The central line of each structural unit lies in a perfect helix, with the centre of the helix located at the centre of yarn cross-section.

(4) The packing of these structural units, in the yarn cross-section is such as to keep their number per unit area, at a direction normal to their axis, constant.

Such an idealised structure will be highly impossible to attain in practice. There will be a difference in tension between the filaments in the core and surface of the structure due to which distortion in structure will take place. Nevertheless if it is assumed that the yarn maintains its rectangular cross-section even after twisting then it can be imagined that on removal of the twisting tension, the most likely condition will be one in which the filaments at the surface of the rectangle will be under tension and core fibres will be slack. The nearest approach to this sort of structure is when a long length of a rectangular array of filaments is twisted at low tension on a twisting apparatus which will allow contraction with twist (twisting apparatus with trolley arrangement).

In continuous spinning, however, the condition is different. The filaments in the yarn will be under varying state of tension depending upon the position they occupy. Those lying at the corner A, B, C, D (Fig. 7.4) of the rectangle will be at a higher tension than the filaments at Y and Y' which are much nearer to the core of the yarn, whereas the filaments at Y and Y' in their turn are at a higher tension that the filaments in the core of the yarn. The filaments will always try to occupy a position where the tension on them will be the least. If this tendency is to be followed by filaments, it could be visualized that the rectangular cross-section could no longer be maintained. In Fig. 7.3(b) A B C D shows in cross-section the rectangular array of filaments before twist is imparted to this assembly. A line passing through O and perpendicular to the plane of the paper is the axis of such an assembly. With O as centre a circle has been drawn so that the circular area is equal to the rectangular area (if the rectangle is of side 'b' and 'd' and if R be the radius of the circle drawn then $bd = \pi R^2$ or

$R = \left(\frac{bd}{\pi}\right)^{1/2}$). If the circle intersects the rectangle at EFGH then all the filaments lying in the area AFED and GBCH will have a higher tension developed in them than the filaments in the area EFGH. Due to this higher tension filaments in the areas AFED and GBCH will move to occupy a position where tension on them is less. It is highly probable that this movement will cause the filaments to re-arrange themselves and occupy positions in the area EJH and GKF and thus giving rise to a cylindrical assembly of filaments. The width of the ribbon and twisting tension have got great influence on the stability of the structure. Wider ribbons will be more accessible to degenerate into a cylindrical assembly. Nevertheless at a very low twist and under low twisting tension twisted ribbon form of structure could be obtained.

7.4 Wrapped Ribbon Structure

The idealised wrapped structure of yarn can be considered to have been formed by wrapping a ribbon of filaments round a cylinder of extremely small diameter. Hence the structure will have a hollow in the centre of the yarn. The further characteristics of such a structure are

(a) The yarn will be uniform along its length and will have a circular cross-section.

(b) The filaments will be following a helical path of constant radius with the centre of the helix lying on the axis of the cylinder round which the ribbon has been wrapped.

Thus the original starting form of such a structure is the rectangular assembly of parallel filaments. Fig. 7.5(a) shows a rectangular assembly of filaments with PQUV being its cross-sectional plane. In Fig. 7.5(b) this ribbon has been shown wrapped into a cylindrical shell structure. If the ribbon has been wrapped in such a way that the plane PQMN becomes parallel to the axis of the yarn then a plane PQRS which was originally perpendicular to the plane PQMN in the ribbon will become parallel to the cross-section of the yarn. The plane PQUV which was perpendicular to the filaments in the ribbon will be inclined to the axis of the yarn in the wrapped form. In Fig. 7.5(b) one turn of twist in the wrapped form has been shown and the path of the filaments has been indicated.

If the ribbon of filaments is assumed to behave as a uniform strip then the wrapped structure as explained above can either be obtained with a gap where edges PQ and RS will lie apart or a jammed structure will be obtained where the two edges will join together. If wrapped structure with a gap be considered then the distance between the edges PQ and RS will not be uniform from surface to the inside of the yarn. There will be a gradual decrease in the gap from outside the yarn to the inside as in Fig. 7.6(a) where a cross-section of the yarn has to be shown. The thickness of the ribbon can be divided into two halves by drawing a line EF as shown in Fig. 7.6(b). If it be considered that the twist introduced in such a yarn is sufficient to make two filaments in the innermost layers QR to touch each other then the filaments at E and F and P and S will not touch each other. In another case if the twist introduced is high enough to make the filaments P and S in the outer layer to touch each other there will be an overlap of filaments in the inner layer as shown in Fig. 7.5(c). If the filaments are free to move, there will be a rearrangement of filaments in the area SPQR due to overlapping and filaments will be pushed into the empty space in the core. If the filaments in the middle layer touch each other, there will be overlap or rearrangement of filaments in the inner layers whereas a gap will exist in the outer layers Fig. 7.5(d).

Since a ribbon of filaments would not behave as a uniform sheet of ribbon and as all the filaments are at a liberty to rearrange themselves, this ideal wrapped structure with hollow in the centre could not be obtained. The filaments in the outer layer of the yarn will be following a longer path length and will develop higher tension in them than the filaments in the inner layers. This will cause the structure to collapse and migration of filaments from outside to the core will occur.

Nevertheless the nearest approach to such an idealised structure can be made by twisting a long length of a ribbon of filaments on a model twister having a trolley arrangement for the contraction in length to take place.

In ring twisting the filaments are fed through a pair of rollers as a result of which flattening of filament bundles take place, and the twist can move near to the nip of the roller. Hence twist is imparted to a flat ribbon of filaments as soon as it emerges out of the delivery rollers. Thus initially a wrapped structure with a very small hollow in the centre might be obtained but due to the higher tension developed in the surface filaments, migration of filaments from the surface to the core will take place and the small hollow from the centre will soon disappear. In cases where an excessive flattening of filament bundle takes place, either due to it being a very heavy denier yarn or due to a very high pressure under the rollers, the width of the ribbon is very large compared with its thickness, distortion in the ribbon take place during twisting. As the filaments on the extreme end of the width of the ribbon have to follow a longer path higher tension is developed in them which causes them to move towards the centre of the yarn. Due to this movement the ribbon folds itself into either a N-shaped or a W-shaped structure before actual wrapping of the ribbon starts. This folding of ribbon will push some filaments into the core of the wrapped structure and hence in the final structure clustering of filaments in the core will be obtained. If in the above case (when excessive flattening occure) the twist can move right upto the roller nip, overlapping of filaments in the ribbon will take place. This in the yarn cross-section would give the appearance of shifting of the core of the yarn from the centre.

7.4.1 Geometry of Ribbon Twisted Yarns

In the discussion of the structure of ribbon twisted yarns it has been pointed out that the cylindrical shell structure does not occur in practice. The shell structure which is obtained at the initial stage collapses and a uniformly packed structure is obtained. Nevertheless, to derive a mathematical relation for the position of a filament in the yarn simplified structures have to be considered. For this purpose wrapped structure with a hollow in the core has been maintained.

To derive a mathematicl expression for the filament position, a ribbon of length 'ℓ', width 'a' and thickness 'b' has been taken to be wrapped into a yarn. The position of a filament in the yarn has been represented in terms of two co-ordinate systems. The original position of a filament in the ribbon before twisting can be represented in terms of the rectangular co-ordinates (X,Y.Z), whereas the final position of the filament in the yarn can be denoted with respect to the cylindrical polar co-ordinates ($r, \phi, \mathfrak{f}$). Fig. 7.7 shows a length of rectangular assembly of filaments ABCD, EFGH. If ABCD and EFGH be the cross-sectional face of such a ribbon of filaments and O and O' be their respective centres then the line joining OO' will be the central axis of the ribbon and is taken as the z-axis. The X and Y axes are taken in the plane of the x-section of the ribbon and intersect each other at any point on the line OO'. If ABCD be the starting end of the ribbon of length 'ℓ' and O be the intersection of the axes X, Y, Z in this cross-sectional face then O is taken as the origin of X, Y,Z axes and is defined by (0,0,0). In the wrapped form a plane containing origin of X,Y,Z co-ordinates is taken as the origin of the r, ϕ, ζ co-ordinates. Fig. 7.8 shows a ribbon of filament wrapped into yarn form and PP' is the axis of this yarn. Let ABCD be the cross-sectional face of the ribbon containing the origin of X,Y,Z, co-ordinates and R be the radius of the yarn lying in the plane of ABCD and passing through the origin O of X,Y,Z axes. If this radius intersects the axes of the yarn PP' at P then P will be defined as the origin of the polar co-ordinate system. The axis of the yarn PP' will represent the ζ-axis. The angular position ϕ of any filament is measured with respect to the radial line PO. Thus, in the yarn the ribbon is considered to be wound round the central axis of the yarn i.e., the z-axis is wound round the ζ-axis and is following a helical path.

The co-ordinates of the point O in terms of the cylindrical polar co-ordinate system will be ($r = R$, $\phi = 0$, $\zeta = 0$). Let the filament in the centre of the ribbon, i.e., the z-axis

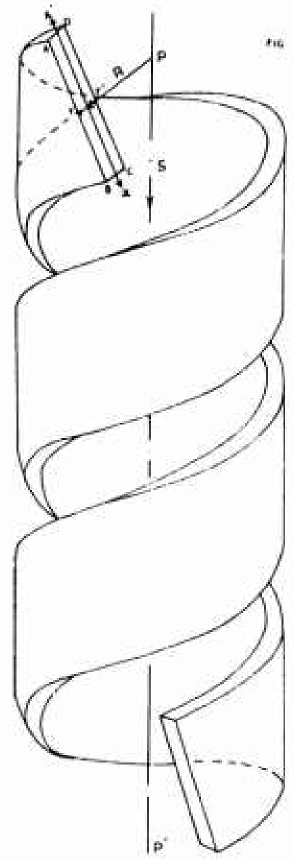


FIG. 7.8

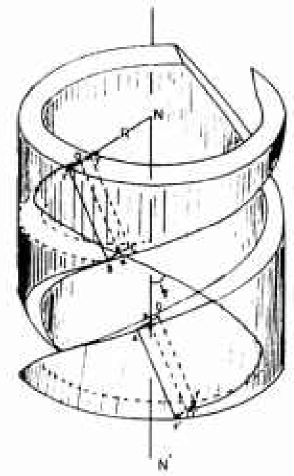
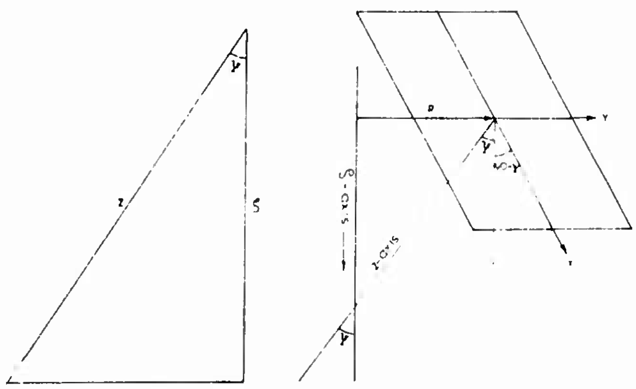
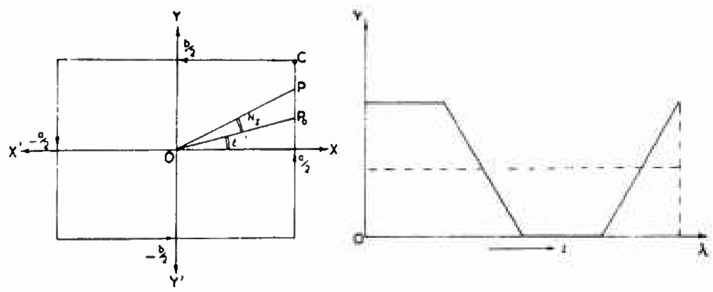


FIG. 7.9



(a) (b)

FIG. 7.11



(a) (b)

FIG. 7.13

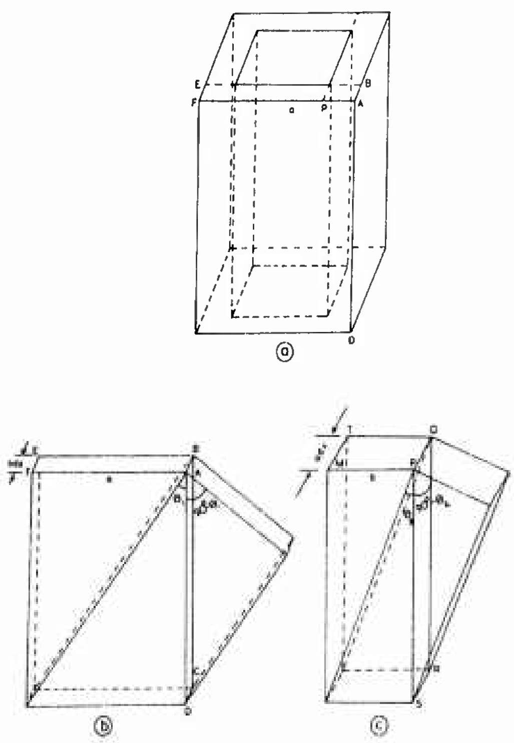


FIG. 7.14

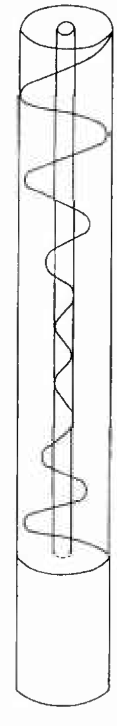


FIG. 7.15

make an angle ψ with a line parallel to the yarn axis or the ζ-axis. Fig. 7.9 shows a section of unit turn of the yarn. On opening such a unit turn of yarn and laying it flat on the plane of the paper as shown in Fig. 7.10, we obtain

$$H^2 = h^2 + 4\pi^2 R^2 \qquad \ldots\ldots \quad (7.7)$$

where H is the length of filament at the axis of the ribbon in a turn of yarn

[illegible] is the length in one turn along the axis of the yarn.

$$\operatorname{Sin} \psi = \frac{2\pi R}{H} \qquad \ldots\ldots \quad (7.8)$$

$$\operatorname{Cos} \psi = \frac{h}{H} \qquad \ldots\ldots \quad (7.9)$$

$$\text{Also} \quad L^2 = h^2 + 4\pi^2(R + y)^2 \qquad \ldots\ldots \quad (7.10)$$

$$\sin\theta = \frac{2\pi(R + y)}{L} \qquad \ldots\ldots \quad (7.11)$$

$$\cos\theta = \frac{h}{L} \qquad \ldots\ldots \quad (7.12)$$

where L is the length of the filament at a distance $(R + y)$ from the yarn axis and θ is the angle this filament makes with the yarn axis.

Then from equations (7.1) and (7.10)

$$z^2 = H^2 + 4\pi^2 y(2R + y) \qquad \ldots\ldots \quad (7.13)$$

$$\text{Also} \quad h = H\cos\psi = L\cos\theta$$

$$\therefore \quad \cos\theta = \frac{H}{L}\cos\psi$$

$$= \frac{H\cos\psi}{\left\{H^2 + 4\pi^2 y(2R + y)\right\}^{1/2}} \qquad \ldots \quad (7.14)$$

$$\text{and} \quad \sin\theta = \frac{1}{L}(2\pi R + 2\pi y)$$

$$= \frac{1}{L}(H\sin\psi + 2y\pi) \qquad \ldots\ldots \quad (7.15)$$

$$= \frac{H\sin\psi + 2\pi y}{\left\{H^2 + 4\pi^2 y(2R + y)\right\}^{1/2}}$$

The position of the filament in the central axis of the ribbon with respect to the cylindrical polar co-ordinate system (r, ϕ, φ) will be

$$r = R \qquad\qquad x = 0$$

$$\phi = 0 \qquad \text{when} \qquad Y = 0$$

$$\varphi = 0 \qquad\qquad z = 0$$

On moving a distance z along the z-axis, the co-ordinate of this point with respect to (X,Y,Z) co-ordinates will be (0,0,z). The same point will occupy a position $r = R$ in the yarn.

When a unit turn has been completed i.e. when a length of H is traversed by the reference point the total rotation of that point round the axis of the yarn is 2π, hence if a length z is traversed the angular rotation $\phi = {}^{2\pi z}/_{H}$.

Again, as the Z-axis makes an angle ψ with a line parallel to ζ-axis as in Fig. 7.11(a). Hence $\zeta = z \cos \psi$.

Hence the point (0,0.z) in the ribbon will occupy a position in yarn denoted by $r = R$, $\phi = \frac{2\pi z}{H}$, $\zeta = z \cos \psi$.

Similarly a point (x,0,0) in the ribbon will occupy a position ($r = R$, $\phi = \frac{x \cos \psi}{R}$, $\zeta = x \sin \psi$) as determined from Fig. 7.11(b)

On traversing a distance z along the Z-axis on the same filament in the ribbon i.e., a point (x,0,z) will have a position ($r = R$, $\phi = \frac{2\pi z}{H} + \frac{x \cos \psi}{R}$, $\zeta = z \cos \psi + x \sin \psi$) in the yarn.

Similarly a point (0,y,0) in the yarn will be ($r = R + y$, $\phi = 0$, $\zeta = 0$) in the yarn and a point (0,y,z) on the same filament in the ribbon will occupy a position ($r = R + y$, $\phi = \frac{2\pi z}{H}$, $\zeta = z \cos \psi$) in the yarn.

A point (x,y,0) in the ribbon will have ($r = (R + y)$, $\phi = \frac{x \cos \psi}{R}$, $\zeta = x \sin \psi$) co-ordinates in the yarn; and any general point (x,y,z) in the untwisted ribbon will occupy a position $[r = (R + y), \phi = \frac{2\pi z}{H} + \frac{x \cos \psi}{R}, \, \theta = z \cos \psi +$ $x \sin \psi]$ in the yarn. When considering unit turn in yarn the co-ordinates of the above general point can be represented as $[r = R + y, \, \phi = \frac{z \sin \psi}{R} + \frac{x \cos \psi}{R}, \, \theta = z \cos \psi + x \sin \psi]$

7.4.2 Effect of spinning twist

The above relations for the position of a filament in the yarn have been arrived at by assuming that in the ribbon all the filaments are lying parallel to each other and the filament in one layer stayed in that particular layer all throughout its length. The filament yarns obtained from the manufacturers have always got certain amount of spinning twist present in them, hence when these filaments are passed through the rollers of a doubling frame for twisting, they are compressed into a ribbon form but it does not have the idealised structure of a ribbon of parallel layer of filaments. Instead, the filaments pass from one layer to the other making a certain angle with the axis of the ribbon depending upon the spinning twist. Thus in the final twisted yarn if a filament is followed along its length, the distance of the filament from the axis of the yarn does not remain constant but it keeps on changing constantly, (as shown in Fig. 7.12), i.e., r changes as y changes and thus migration of filaments will be noticed. The values of θ and ϕ also change due to the spinning twist.

If λ be the length of ribbon per unit turn of spinning twist, then turns per unit length of spinning twist $= \frac{1}{\lambda}$. Observing the path of a filament from the cross-sectional face of the yarn, it will be seen that the filaments are following a path of a wave pattern as shown in Fig. 7.13(a) and (b), where the arrows show the direction of movement of the reference point.

If the initial position of a particular filament is P_0 (Fig. 7.13(a)), so that it makes an angle ϵ with the X-axis then on moving a distance z along the Z-axis let the position of the filament become P. Then if uniform rotation is assumed, the number of turns through which it will have rotated will be $\frac{1}{\lambda} \cdot z$.

$$\text{i.e.} \quad = \frac{2\pi}{\lambda} \cdot z \text{ degrees} \quad = Nz \qquad \text{where } N = \frac{2\pi}{\lambda}$$

Hence the total angle this point P will be making with the abscissae will be $= (Nz + \epsilon)$. The displacement of the filament along the X or Y axis as it advances along the Z-axis is shown in Fig. 7.13(b). The curve will indicate that any form of spinning twist will repeat after λ. Such a displacement curve can be analysed by Fourier's Theorem and displacement along X or Y axis is given by -

$$y = y_0 + \sum_{i=1}^{i=\infty} A_i \cos iNz + \sum_{i=1}^{i=\infty} B_i \sin i\,Nz$$

$$x = x_0 + \sum_{i=1}^{i=\infty} C_i \cos iNz + \sum_{i=1}^{i=\infty} D_i \sin iNz$$

Since $\xi = z \cos \psi + x \sin \psi$

$$\therefore \quad \xi = z \cos \psi + \left[x_0 + \sum_{i=1}^{i=\infty} C_i \cos iNz + \sum_{i=1}^{i=\infty} D_i \sin iNz \right] \sin \psi$$

and $r = R + y$

$$= R + y_0 + \sum_{i=1}^{i=\infty} A_i \cos iNz + \sum_{i=1}^{i=\infty} B_i \sin iNz \qquad \ldots\ldots (7.16)$$

We know that when z increases by λ i.e. when one full turn has been completed

(i) y returns to same value

(ii) x returns to same value

(iii) ξ increases by $\lambda \cos$

and (iv) r returns to same value.

$$\therefore \quad r = R = y_0 + \sum_{i=1}^{i=\infty} E_i \cos i \frac{2\pi}{\lambda \cos \psi} \xi.$$

$$+ \sum_{i=1}^{i=\infty} F_1 \sin i \frac{2\pi}{\lambda \cos \psi} \xi. \qquad \ldots\ldots (7.17)$$

Thus migration period of a filament will be $\lambda \cos \psi$ and the value of ξ for any filament when one cycle of migration has taken place is given by

$$\xi = \lambda \cos \psi + \left[x_0 + \sum_{i=1}^{i=\infty} C_i \cos iN\lambda + \sum_{i=1}^{i=\infty} D_i \sin i\lambda \right] \sin \psi \qquad \ldots\ldots (7.17a)$$

To obtain the radial positions of any particular filament along the length of the yarn a simpler analysis could be adopted. The important step of this analysis would be to determine the intercept made by that particular filament on the Z-axis as it passes from one face of the ribbon to the other. Two assumptions are to be made for this purpose.

(a) There is equal density of packing of filaments in the yarn, irrespective of the face of the ribbon they are passing through.

(b) Considering a length of ribbon of one turn of twist, the number of filaments passing each section along the width or thickness of the ribbon must be equal.

The above condition is true if the cross-sectional area of the element on the wider face is equal to the cross-sectional area of the element in the narrower face. If 'a' is the width of the wider face and 'b' is the width of the narrower face (Fig. 7.4a) then the thickness of the element of face 'a' must be equal to bdx and that of face 'b' will be adx

So that $a\,(bdx) = b\,(adx)$.

Considering each element separately

Let the filaments make an angle θ_1 and θ_2 with a line parallel to the axis of the ribbon in the element with width 'a' and 'b' respectively and if M be the total number of filaments crossing each element of area ABCD and PQRS then from Fig. 7.14(b) and (c) the number of filaments crossing per unit area perpendicular to the axis of filaments in the face

$$ABCD = \frac{M}{bdx.\ \lambda \sin \theta_1}$$

Similarly the number of filaments crossing per unit area

perpendicular to the filaments in face

$$PQRS = \frac{M}{adx\ .\ \lambda \sin \theta_2}$$

Then from our assumption of constant packing density in a plane perpendicular to the filament axis:

$$\frac{M}{bdx\ \lambda \sin \theta_1} = \frac{M}{adx\ \lambda \sin \theta_2}$$

$$\text{or} \quad \frac{\text{Sin}\ \theta_1}{\text{Sin}\ \theta_2} = \frac{a}{b} \qquad \ldots\ldots \quad (7.18)$$

Since for spinning twists the angles θ_1 and θ_2 are very small hence $\text{Sin}\ \theta_1$ and $\text{Sin}\ \theta_2$ can be replaced by $\tan\theta_1$ and $\tan\theta_2$.

$$\therefore \quad \frac{\tan \theta_1}{\tan \theta_2} = \frac{a}{b} \qquad \ldots\ldots \quad (7.19)$$

$$\text{or} \quad a \cot \theta_1 = b \cot \theta_2 \qquad \ldots\ldots \quad (7.20)$$

Considering unit turn of twist

$$a \cot \theta_1 + b \cot \theta_2 = \lambda/_2 \qquad \ldots\ldots \quad (7.21)$$

$\therefore$ From equations (7.20) and (7.21)

$$\left.\begin{aligned} \cot \theta_2 &= \frac{\lambda}{4b} \\ \text{and} \quad \cot \theta_1 &= \frac{\lambda}{4a} \end{aligned}\right) \qquad \ldots\ldots \quad (7.22)$$

Since the value of z for the wider face of the ribbon is

$$z_w = a \cot \theta_1$$

$$= a \cdot \frac{\lambda}{4a} = \frac{\lambda}{4}$$

and the value of z for the narrower face of the ribbon is

$$z_n = b \cot \theta_2$$

$$\therefore \quad z_n = b \cdot \frac{\lambda}{4b} = \frac{\lambda}{4}$$

This signifies that equal intercepts are made on the Z-axis of the ribbon by a filament due to its spinning twist.

To examine analytically how the position of a filament changes as it advances along the z-axis, when uniform packing density of filaments is considered in the ribbon we refer to Fig. 7.13(a). If C be the starting position of a filament on the surface of the yarn then :

<u>Case A :</u>

When filament moves from $z = 0$ to $z = \lambda/4$, along the wider face of the ribbon

$y = + \frac{b}{2}$ (remains constant during first quarter of a turn).

$x = + \frac{a}{2} + z\,(-a)/(\frac{\lambda}{4})$ (the value of x changes and it moves from position C to D).

Case B :

When the filament advances from $z = \frac{\lambda}{4}$ position to $z = \lambda/2$, the filament is crossing the narrow face of the ribbon and the value of x remains constant during this traverse.

$$x = -\frac{a}{2}$$

but y varies and is $= +\frac{b}{2} + (z - \frac{\lambda}{4}) \left\{ (-b)/\frac{\lambda}{4} \right\}$

Case C :

When the filament advances further than $z = \frac{\lambda}{2}$, it crosses over to the wider face of the ribbon once again but this time on the rear side of the ribbon and again the position of filament along the y-direction remains constant as long as it traverses this face but position along x-direction varies. Hence from

$z = \frac{\lambda}{2}$ to $z = \frac{3\lambda}{4}$

$$y = -\frac{b}{2}$$

$$x = -\frac{a}{2} + (z - \frac{\lambda}{2})(a/\frac{\lambda}{4})$$

Case D :

When the filament position along the z-axis increases more than $\frac{3\lambda}{4}$ the filament crosses over to the narrower face of the ribbon to complete one full turn.

Hence from $z = \frac{3\lambda}{4}$ to $z = \lambda$

$$x = +\frac{a}{2}$$

and $y = -\frac{b}{2} + (z - \frac{3\lambda}{4})(b/\frac{\lambda}{4})$

The same pattern will be seen repeating in the next turn and the turn after and so on. Hence on generalising it can be written as:-

TABLE 7.1

VALUES OF z	VALUES OF y	VALUES OF x	RADIAL POSITION r
$i\lambda$ to $i\lambda + \frac{\lambda}{4}$	$+\frac{b}{2}$	$+\frac{a}{2} - \frac{4a(z-i\lambda)}{\lambda}$	$R + \frac{b}{2}$
$i\lambda + \frac{\lambda}{4}$ to $i\lambda + \frac{\lambda}{2}$	$+\frac{b}{2}+(z-i\lambda-\frac{\lambda}{4})(-\frac{4b}{\lambda})$	$-\frac{a}{2}$	$R+\frac{b}{2}-\frac{4b}{\lambda}(z-i\lambda-\frac{\lambda}{4})$
$i\lambda+\frac{\lambda}{2}$ to $i\lambda+\frac{3\lambda}{4}$	$-\frac{b}{2}$	$-\frac{a}{2}+\frac{4a}{\lambda}(z-i\lambda-\frac{\lambda}{2})$	$R - \frac{b}{2}$
$i\lambda+\frac{3\lambda}{4}$ to $i\lambda+\lambda$	$-\frac{b}{2}+(z-i\lambda-\frac{3\lambda}{4})(\frac{4b}{\lambda})$	$+\frac{a}{2}$	$R-\frac{b}{2}+\frac{4b}{\lambda}(z-i\lambda-\frac{3\lambda}{4})$

In order to determine the radial position of a filament in the yarn as a length is traversed along the yarn axis, the terms containing z can be replaced by $\mathcal{J}$ by using the equation:

$$\mathcal{J} = z \cos\psi + x \sin\psi$$

Table 7.2 gives the radial position for different values of $\mathcal{J}$ along the yarn axis, and evaluation of these have been shown in Appendix 1.

TABLE 7.2

FOR VALUES OF ρ	VALUES OF r
$i\lambda(\cos\psi - \frac{4a}{\lambda}\sin\psi) + (\frac{a}{2} + 4ai)\sin\psi$ To $(i\lambda + \frac{\lambda}{4})(\cos\psi - \frac{4a}{\lambda}\sin\psi) + (\frac{a}{2} + 4ai)\sin\psi$	$R + \frac{b}{2}$
$(i\lambda + \frac{\lambda}{4})\cos\psi - \frac{a}{2}\sin\psi$ To $(i\lambda + \frac{\lambda}{2})\cos\psi - \frac{a}{2}\sin\psi$	$R + \frac{b}{2} - \frac{4b}{\lambda}\left\{(\rho + \frac{a}{2}\sin\psi)\sec\psi - i\lambda - \frac{\lambda}{4}\right\}$
$(i\lambda + \frac{\lambda}{2})(\cos\psi + \frac{4a}{\lambda}\sin\psi) - (\frac{5a}{2} + 4ai)\sin\psi$ To $(i\lambda + \frac{3\lambda}{4})(\cos\psi + \frac{4a}{\lambda}\sin\psi) - (\frac{5a}{2} + 4ai)\sin\psi$	$R - \frac{b}{2}$
$(i\lambda + \frac{3\lambda}{4})\cos\psi + \frac{a}{2}\sin\psi$ To $(i\lambda + \lambda)\cos\psi + \frac{a}{2}\sin\psi$	$R - \frac{b}{2} + \frac{4b}{\lambda}\left\{(\rho - \frac{a}{2}\sin\psi)\sec\psi - i\lambda - \frac{3\lambda}{4}\right\}$

Thus if a rectangular assembly of filaments having spinning twist (as shown in Fig. 7.7 where the filaments make equal intercepts on either face of the ribbon) be twisted to give a wrapped structure, a filament that was initially on the surface will follow a path as shown in Fig. 7.12. Let λ be the length of filament per unit turn of spinning twist and P be the period of migration.

In the final wrapped yarn if the helix angle of filaments on the surface be ψ_1 and the helix angle of the filaments in the inner layer be ψ_2 , then the contraction factor of the yarn will be given by

$$\frac{\lambda}{P} = \frac{1}{2}(\sec \psi_1 + \sec \psi_2) \qquad \ldots\ldots \quad (7.23)$$

In case of a collapsed wrapped structure the contraction factor will be given by

$$\frac{\lambda}{P} = \frac{1}{2}(\sec \psi_1 + 1) \qquad \ldots\ldots \quad (7.24)$$

which is similar to the contraction factor formula given by Morton and Hearle. (17).

The angular rotation ϕ of the filament round the axis of the yarn will be given by

$$\phi = \frac{z \sin \psi}{R} + x \frac{\cos \psi}{R}$$

Since $\rho = z \cos\psi + x \sin\psi$

$$\therefore \quad z = [\rho - x \sin \quad] \sec\psi$$

Hence
$$\Phi = [\rho - x \sin\psi] \sec\psi \cdot \frac{\sin\psi}{R} + \frac{x \cos\psi}{R}$$

$$= \rho \frac{\tan\psi}{R} - \frac{x}{R} \cdot \frac{\sin^2\psi}{\cos\psi} + \frac{x}{R} \cos\psi$$

$$= \rho \frac{\tan\psi}{R} - \frac{x}{R} \cos\psi \left(\frac{\sin^2\psi}{\cos^2\psi} - 1 \right)$$

$$= \rho \frac{\tan\psi}{R} - \frac{x}{R} \cos\psi \left(\frac{\sin^2\psi - \cos^2\psi}{\cos^2\psi} \right)$$

$$= \rho \frac{\tan\psi}{R} + \frac{x}{R} \frac{\cos 2\psi}{\cos\psi} \qquad \ldots\ldots \quad (7.25)$$

CHAPTER 8

STUDY OF YARNS TWISTED ON COMMERCIAL MACHINES

8.1 Introduction

In all the earlier theories yarn was assumed to have been formed by twisting a cylindrical assembly of parallel filaments. The twisting processes of commercial yarns reveal that, when fed through a pair of delivery rollers, fibres and filaments are presented in a flattened ribbon form at the roller nip and twist is imparted to such a ribbon of fibres. When twisted this way, it would be expected that fibres or filaments going to form the surface layer of the yarn would completely cover the fibres or filaments on the other surface of the ribbon. Untwisting of colour coated yarns described in Chapter 2 showed coloured and uncoloured portions alternating in yarns. At this stage, when the presence of a ribbon structure has been established it was thought that further experimental proof of the presence of such a structure is necessary. If the problem is approached in the reverse way to that in Chapter 2, i.e., instead of coating the final yarn with a colour and untwisting them, making up a yarn by using two layers of coloured and uncoloured fibres or filaments, then the positions occupied by the coloured and uncoloured fibres in the final yarn will indicate the form of twisting of the yarn. For this purpose a controlled feed of coloured fibres during

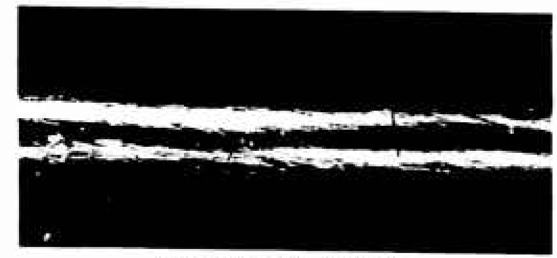
FIG. 8.2

WRAPPED STRUCTURE IN FILAMENT YARNS.

(Cross-Sectional View.)

FIG. 8.3 WRAPPED STRUCTURE IN STAPLE FIBRE ROVINGS.

(Cross-Sectional View)

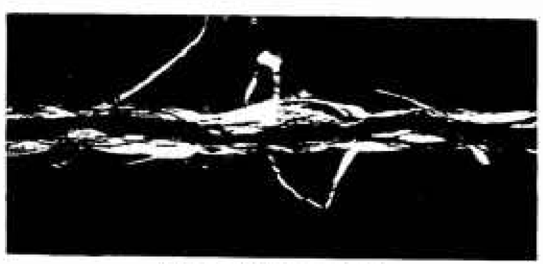


(Longitudinal View.)

FIG. 8.4

WRAPPED STRUCTURE IN STAPLE YARNS.

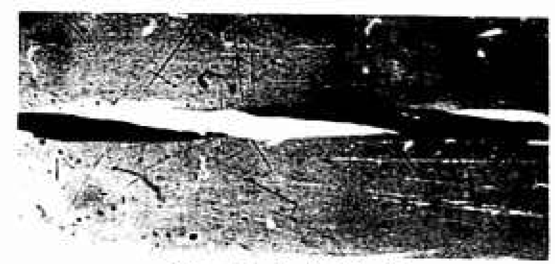
(Cross-Sectional View.)



(Longitudinal View.)

FIG. 8.1 CYLINDRICAL TWISTING OF FILAMENT YARNS.

(Cross-Sectional View)



(Longitudinal View.)

twisting was maintained. Cross-sections of the yarns produced are reported in this chapter.

8.2 Experimental

8.2.1 Cylindrically Twisted Yarns

It was thought that if cylindrically twisted yarn could be produced such that the filaments in the yarn were half coloured and half uncoloured then any other form of twisting could be distinguished by comparing them with this yarn. Since it is extremely difficult to obtain a cylindrical assembly of parallel filaments for twisting on any commercial spinning frame, a small attachment, similar to the one explained on page 95 to produce cylindrical assembly of filaments, was made. The diameter of the hole drilled in the Perspex bar was chosen to give a compact cylindrical assembly of filaments. This attachment was mounted in front of the delivery rollers of a roving frame. Thus although the filaments were flattened by the rollers of the frame, they were made cylindrical before twist was imparted. The coloured filaments were passed through the top half of the hole and the uncoloured filaments from the bottom half. A fine needle passing across the diameter of the hole was placed in the middle of the two strands of filaments in order to stop the twist running through the hole and going up to the roller nip. This made sure that twist was being imparted to a cylindrical assembly of filaments.

(Fig. 8.1).

8.2.2 Ribbon Twisting

(a) Filament yarns. - The ideal condition would have been to have a single ribbon of filaments with the top layer coloured and the bottom layer uncoloured but as it is difficult to obtain such a strand two separate strands, one coloured and another uncoloured, were fed one over the other through the delivery rollers of the twisting machine. Because of the pressure under the rollers the two strands tended to lie side by side rather than pass exactly on top of the other. For this reason a careful control of strands was necessary. The filaments were twisted on a roving frame using only the front pair of rollers. A rectangular shaped trumpet was used to feed the two strands one over the other and right up to the roller nip. (Fig. 8.2)

(b) Staple Fibre Yarns.

(i) Twisted Rovings - Due to a certain amount of twist present in the rovings, feeding of two coloured rovings, one on the top of the other, at the ring frame was not possible. The rovings tend to slide and lie side by side under the rollers. Also at the ring spinning frame the count of the material is so fine that it is difficult to handle the material and thus a ribbon of fibres having coloured fibres on one of its face could not be produced. For these reasons it was decided to produce heavier yarns at the fly-frame or the roving frames where a heavier count of material could be processed and hence desirable control of fibres was possible. Two slivers of equal hanks (one of them coloured) were passed through the silver guide at the back of the drafting rollers. The sliver guide used was of rectangular trumpet shape and the slivers were passed through this trumpet one over the other. On drafting these slivers a thin ribbon of fibres having two coloured layers were obtained. Twist was imparted to this ribbon by the flyers. (Fig. 8.3).

(ii) Twisted Yarns - As mentioned before, on a ring spinning frame where rovings have to be fed to obtain the final yarn, it was very difficult to produce a ribbon of filaments that would have two layers of coloured fibres one over the other. In certain spinning frames where yarn can be spun directly from the sliver (Japanese O.M. Super draft ring frames), it was possible to feed the slivers in the desired manner. This machine uses two separate zones for drafting the sliver and each zone is preceded by a trumpet having a rectangular slot to feed the sliver in each zone. This provides a perfect control for keeping a layer of coloured fibres over the uncoloured layer. In our experiment a coloured sliver and an uncoloured sliver, each of 0.37 hank, were used to produce a 30^{s} yarn (cotton count). Thus the draft in the machine was approximately 160.

In producing all these yarns viscose rayon filaments and staple fibres were used. These yarns were embedded in 'ceemar' resin. The refractive index of cumar resin is the same as that of viscose rayon and hence the uncoloured filaments were rendered transparent and only the coloured filaments and fibres could be seen. On solidifying the resin becomes hard like Perspex, and is then ground with emery and polished to give a smooth shining surface. The yarns were then examined under microscope. To observe clearly the position of coloured and uncoloured fibres or filaments in the yarn, polarized light was used. The cross-sectional and longitudinal views of the yarns were photographed and are shown in Figure 8.4.

8.3 Results

(a) Cylindrically Twisted Yarns: The longitudinal view of the photograph shows that the white and coloured fibres are alternating along its length, and in the cross-sectional view the coloured layers have stayed as two separate halves in the yarn. This sort of result would be expected if a solid cylinder with its two halves differently coloured was twisted around its central axis.

Since yarns are not twisted under similar conditions as mentioned above, such a form may not be obtained in commercial yarns. Nevertheless some yarns might have a very near approach to this type of structure and hence can be regarded as cylindrically twisted yarns.

(b) Ribbon Twisted Yarns:

(i) Filament Yarns - The cross-sectional view of the filament yarn (Fig. 8.2) twisted on the roving frame shows that all the coloured filaments have gone to the centre of the yarn whereas the white filaments have formed the surface layer. If a ribbon of filaments was twisted so that wrapping would occur, then such a structure will arise. When delivered through a pair of rollers, the filaments get flattened and hence on twisting give rise to wrapped ribbon structure. The cross-section shows that there is a cluster of white filaments near the centre of the yarn. This might have been due to the folding of the ribbon during twisting.

(ii) Staple fibre yarns - The cross-sectional and longitudinal views of the roving twisted on the flyer frame indicate the presence of wrapped ribbon structure. It can be seen that all the coloured fibres have gone to the centre of the yarn and white fibres are wrapped round them. The cross-section further shows that the coloured filaments are nearer to the surface on one end of the yarn than on the diametrically opposite end. This might be due to the ribbon of fibres wrapping round and jamming at that particular end.

The cross-section of the yarn twisted on ring spinning frame shows that although almost all the coloured fibres are on the surface of the yarn yet the white fibres are not completely surrounded by them. Some of the white fibres have appeared on the surface whereas a few coloured fibres have moved to the inner layers of the yarn. This will be expected to happen if fibre migration takes place from the surface to the core of the yarn, thus disrupting an ideal wrapped ribbon structure.

8.4 Conclusions

The above studies of the commercial yarns further substantiate that staple fibre and filament yarns when twisted on a spinning frame, where the material is fed by a pair of rollers, will give rise to wrapped ribbon type of structure. The coarse yarns at low twists will give purely a wrapped form but as the twist in the yarn is increased, migration of fibres and filaments will take place causing the structure to deviate from the ideal wrapped structure.

CHAPTER 9

CONCLUSIONS

9.1 General Observations

The theoretical and experimental analysis for the forms of yarn show that when a flat ribbon of fibres or filaments is twisted two distinct forms appear,

(a) Twisted Ribbon Form

(b) Wrapped Ribbon Form.

Alternatively a yarn may twist as a cylindrical bundle. Figure 9.1 has been drawn to illustrate clearly the difference between the cylindrical form, twisted ribbon form and wrapped ribbon form of structures. The shaded portions have been drawn to indicate the position of a group of filaments before and after twisting. Thus in case of twisted cylindrical structure and twisted ribbon structure the rotation of the yarn has taken place around the axis of the cylinder and axis of the ribbon respectively, whereas in case of wrapped ribbon structure the axis of rotation does not coincide with the axis of the ribbon. As the name implies the ribbon is wrapped round another axis which becomes the axis of the yarn. This causes the filaments on one layer to go inside the yarn structure as shown in Fig. 9.1(c).

9.2 Theoretical

9.2.1 Mechanics of Twisting

To determine what form would occur the minimum energy theory was applied. The theory indicated that at low twists the strain energy in the twisted ribbon form is small and hence is preferred but at higher twists the energy stored in the wrapped ribbon form is smaller than the twisted ribbon form and hence the wrapped ribbon structure will be preferred. The theoretical analysis further showed that for wider ribbons the wrapped ribbon form of structure is obtained at lower twists than for narrow ribbons. The theoretical analysis also indicated that the minimum energy in a wrapped ribbon structure lies in the region of little overlap of edges of the ribbon. Due to the finite thickness of ribbons, overlap is not always possible but a jammed wrapped structure is obtained.

9.2.2 Migration Due to Spinning Twist

In a ribbon of filaments where all the filaments are considered to be lying parallel to each other, then if no tension difference between the filaments during twisting is considered, the migration of filaments will be zero. But the presence of spinning twist in the yarn will show migration of filaments in the final wrapped structure. The frequency of migration (number of reversals per unit length) will be approximately equal to the spinning twist.

The period of migration will be slightly smaller than the length of spinning twist, due to contraction in length.

Fig. 9.2(a) shows the migration pattern of a filament due to spinning twist. The radial distances of the filament along the yarn axis have been calculated assuming the ideal wrapped ribbon structure with a hollow in the centre. This final yarn is considered to have been formed by twisting a length of yarn, having one turn of spinning twist, through thirty turns. The corrected helix envelope profile (which is a plot of Y along the axial distance ζ , where $Y = (\frac{\rho}{R})^2$, ρ being the radial distance of filament from the yarn axis and R is the yarn radius) shows a migration pattern as shown in Fig. 9.3(a). It can be seen that the radius of a filament initially on the surface of the yarn remains constant for a certain period. Then it decreases uniformly for another period until it reaches a value where its radial distance from the yarn axis is equal to the inner radius of the yarn. This radial distance stays constant for another period and then it starts increasing until the filament reaches the outer surface of the yarn and thus completes the full cycle of migration. These periods, during which the radius values remain constant, decrease or increase at a steady rate, are all equal and have values equal to $\lambda/4$ where λ is the length in one turn of spinning twist. In Fig. 9.2(b) the decrease in radial distance of a filament from the yarn centre has been shown. The O position in Fig. 9.2(b) corresponds to the B position in Fig. 9.2(a), 1 in Fig. 9.2(b) is the radial distance after half a turn and 2 position is the radial distance after one full turn in the yarn. Thus 15 will correspond to the C position in the yarn. When moving from D to E in Fig. 9.2(a) the radial distances will increase from 15 to 0 in Fig. 9.2(b).

This in principle will show a complete ideal migration as indicated by the dotted line in Fig. 9.3(b). The hollow centred yarn will be unstable and will collapse to give a yarn with no hollow in the centre. If no major distortion in yarn is assumed, due to this collapsing the ideal migration pattern shown by the filament will be indicated (dotted line) in Fig. 9.4., where the filament is seen migrating from the surface to the core and to the surface again.

If the migration due to tension difference in filaments is now considered this will be expected to show some sort of secondary migration superimposed on the primary migratory cycle due to spinning twist as shown in Fig. 9.4. The contraction in length will be given by

$$(\lambda - P) = P(\sec \psi - 1)$$

where λ is the length in one turn of spinning twist and P is the period of migration and ψ is the helix angle.

9.3 Experimental

9.3.1 Model Yarns. The experimental studies on model yarns indicate the presence of twisted ribbon form of yarn at a low twist. This form is very unstable and degenerates into a cylindrical form. When twisted at constant tension the wrapped ribbon structure with a hollow in the centre is obtained (twisted on a sliding trolley type twister). On continuous twisting type machines where the twisting conditions are more like in conventional spinning frames, the wrapped ribbon form of yarn, with a hollow in the centre, is not obtained. The yarn collapses into a compact bundle of filaments.

When twisted on a continuous twisting type machine it was further observed that the ribbon of filaments does not fold into an ideal wrapped structure, instead certain amount of distortion takes place during twisting. The ribbon first folds into an N-shaped or W-shaped assembly of filaments and then wrapping takes place. Folding of ribbons pushes some of the filaments to the core of the yarn and hence a wrapped structure with a hollow in the centre is not formed. When the ribbon width is excessively large overlapping of filaments at the edges of the ribbon occurs. This gives the appearance of the core of the yarn slightly displaced off centre.

9.3.2 Actual Yarns

Experiments on commercial yarns (staple and filament), twisted on spinning frames having roller delivery, show that the final yarn has the wrapped ribbon structure with collapsed centre. Examination of the cross-sections of filament yarns shows that the heavy denier yarns (e.g. 400 den. yarns etc.) have a cluster of filaments in the centre of the yarn. The 800 denier yarns give the impression of the core being shifted slightly off centre. The fine denier yarns (100 den. etc.) show uniform packing of filament in the yarn cross-sections. All these observations show that the forms of actual yarns can be indicated by the model yarns. When spreading of filaments is large the wrapped ribbon form of structure is obtained.

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APPENDIX I

$$\xi = z \cos\psi + x \sin\psi$$

(A) When z varies from $z = i\lambda$ to $i\lambda + \dfrac{\lambda}{4}$

$$= z \cos\psi + \left\{ \frac{a}{2} - \frac{4a(z - i\lambda)}{\lambda} \right\} \sin\psi$$

$$= z \cos\psi - \frac{4az}{\lambda} \sin\psi + \left(\frac{a}{2} + 4ai\right) \sin\psi$$

$$= z \left(\cos\psi - \frac{4a}{\lambda} \sin\psi\right) + \left(\frac{a}{2} + 4ai\right) \sin\psi$$

Hence value of ξ changes from

$$\xi = i\lambda\left(\cos\psi - \frac{4a}{\lambda} \sin\psi\right) + \left(\frac{a}{2} + 4ai\right) \sin\psi$$

to

$$\xi = \left(i\lambda + \frac{\lambda}{4}\right)\left(\cos\psi - \frac{4a}{\lambda} \sin\psi\right) + \left(\frac{a}{2} + 4ai\right) \sin\psi$$

(B) When filament advances from $z = \left(i\lambda + \dfrac{\lambda}{4}\right)$ to $\left(i\lambda + \dfrac{\lambda}{2}\right)$

ξ advances from

$$\xi = \left(i\lambda + \frac{\lambda}{4}\right) \cos\psi - \frac{a}{2} \sin\psi$$

to

$$\xi = \left(i\lambda + \frac{\lambda}{2}\right) \cos\psi - \frac{a}{2} \sin\psi$$

(C) When the value of z changes from $z = \left(i\lambda + \dfrac{\lambda}{2}\right)$ to $\left(i\lambda + \dfrac{3\lambda}{4}\right)$

$$\xi = z \cos\psi + \left\{ -\frac{a}{2} + \frac{4a}{\lambda}\left(z - i\lambda - \frac{\lambda}{2}\right) \right\} \sin\psi$$

$$= z\cos\psi + \frac{4az}{\lambda}\,.\,\sin\psi - \left(\frac{5a}{2} + 4ai\right)\sin\psi$$

$$= z\left(\cos\psi + \frac{4a}{\lambda}\sin\psi\right) - \left(\frac{5a}{2} + 4ai\right)\sin\psi$$

Hence the value of ζ advances from

$$\zeta = \left(i\lambda + \frac{\lambda}{2}\right)\left(\cos\psi + \frac{4a}{\lambda}\sin\psi\right) - \left(\frac{5a}{2} + 4ai\right)\sin\psi$$

to

$$\zeta = \left(i\lambda + \frac{3\lambda}{4}\right)\left(\cos\psi + \frac{4a}{\lambda}\sin\psi\right) - \left(\frac{5a}{2} + 4ai\right)\sin\psi$$

(D) When z varies from $z = \left(i\lambda + \frac{3\lambda}{4}\right)$ to $z = (i\lambda + \lambda)$

ζ varies from

$$\zeta = \left(i\lambda + \frac{3\lambda}{4}\right)\cos\psi + \frac{a}{2}\sin\psi$$

to

$$\zeta = (i\lambda + \lambda)\cos\psi + \frac{a}{2}\sin\psi$$